MSE 510
Final Examination
May 5, 2015

1. Single Variable Nonlinear Optimization

Download the data located at
http://utkstair.org/clausius/docs/mse510/data/mse510_xm02_p01.txt
This data represents the results of a set of experiments intended to measure the variance in the particle size of a crystallization process. The data is essentially a histogram with the first column corresponding to variance, *x*, and the second column corresponding to probability, *f*. From theory this data should follow the chi-squared distribution, given by

$$f_{\chi^2}(x;v) = \begin{cases} \frac{1}{2^{v/2}\Gamma(v/2)} \mathrm{x}^{v/2-1}\mathrm{e}^{-x/2} & \text{for } x > 0 \\ 0 & \text{elsewhere} \end{cases} \quad (1)$$

The chi-squared distribution has one parameter, *v*, the degrees of freedom. ($\Gamma(x)$ is the gamma function, an intrinsic function in Matlab, gamma(x).) Perform a single variable nonlinear optimization in order to fit the data to this model. Report the optimal value of *v*. As a good initial guess, consider that the population mean of the chi-squared distribution is *v*.

2. Single Non-Linear Parabolic PDE

The one-dimensional heat equation can describe heat transfer in a material with both heat conduction and radiative heat loss.

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p}\frac{d^2T}{dz^2} - \frac{\varepsilon\sigma S}{\rho C_p}\left(T^4 - T_s^4\right)$$

where the following variables [with units] are given as

- temperature in the material T [K]
- surrounding temperature $T_s = 77$ [K]
- axial position along material z [m]
- thermal conductivity $k = 401$ [J/K/m/s] (for Cu)
- mass density $\rho = 8960$ [kg/m^3] (for Cu)
- heat capacity $C_p = 384.6$ [J/kg/K] (for Cu)
- Stefan–Boltzmann constant $\sigma = 5.670373x10^{-8}$ [J/s/m^2/K^4]
- gray body permittivity $\varepsilon = 0.15$ (for dull Cu)
- surface area to volume ratio $S = 80$ [m^{-1}] (for a cylindrical rod of diameter 0.05 m)

Problem 2 continued on reverse side.

2. Single Non-Linear Parabolic PDE (continued)

A cylindrical Cu rod of diameter 0.05 m and length 0.5 m is initially at $T(z,t=0)=800$ K. One end of the rod is maintained at $T(z=0,t)=900$ K. The other end of the rod is insulated, $\left.\frac{dT}{dz}\right|_{z=0.3}=0$ K/m.

(a) Plot the transient behavior.
(b) Find the approximate steady-state temperature in the material at z=0.5 m.

3. ODE Boundary Value Problem

Consider the following boundary value problem, which represents the steady state profile in Problem 2.

$$0=\frac{k}{\rho C_p}\frac{d^2T}{dz^2}-\frac{\varepsilon\sigma S}{\rho C_p}\left(T^4-T_s^{\,4}\right)$$

with the boundary conditions

$$T(z=0)=T_o=900\ \text{K}$$
$$\frac{dT}{dz}(z=0.5)=T_f'=0\ \text{K/m}$$

where all of the parameters are given in problem 2.

(a) Convert this single second-order ODE, to a system of two first-order ODEs.
(b) Plot the solution.
(c) What is the temperature gradient at z = 0?
(d) What is the temperature at z = 0.5?