Midterm Examination
Administered: Wednesday, October 10, 2001

Problem (1)

Consider the 2x2 matrix: $\underline{\underline{A}} = \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix}$

(a) Find the eigenvalues.
(b) Find the normalized eigenvectors.
(c) If $a_{11} = a_{22}$, does $\lambda_1 = \lambda_2$? Why or why not?
(d) If $a_{11} = a_{22}$, does $\underline{w}_1 = \underline{w}_2$? Why or why not?

Problem (2)

Consider the system of nonlinear algebraic equations:

$\sin(x_1) + x_2 = 0$ and $x_1^2 + x_2^2 = 1$

where x_1 has units of radians.
(a) If we are going to solve this system using multivariate Newton-Raphson, we need the Jacobian and the residual. Determine them.
(b) For an initial guess of $(x_1, x_2) = (\frac{1}{2}, -\frac{1}{2})$, Evaluate the Jacobian and residual and the next estimate of the solution using multivariate Newton-Raphson.

Problem (3)

If the determinant of an nxn matrix $\underline{\underline{A}}$ is 1.0, what can you say about:
(a) the rank of the matrix
(b) the linear dependence of the equations which form the matrix
(c) the inverse of the matrix
(d) the eigenvalues of the matrix
(e) the eigenvectors of the matrix
(f) the number of solutions to $\underline{\underline{A}}\underline{x} = \underline{b}$

Problem (4)

Consider the ordinary differential equation, where f is in general a nonlinear function, and the following conditions:

$$\frac{d^3y}{dx^3} = f\left(x, y, \frac{dy}{dx}, \frac{d^2y}{dx^2}\right)$$

$$y(x = x_o) = y_o \qquad \left.\frac{dy}{dx}\right|_{x=x_o} = y'_o \qquad \left.\frac{dy}{dx}\right|_{x=x_f} = y'_f$$

Provide an algorithm for numerically obtaining a solution to this ordinary differential equation. Explain any transformations or recastings of the equations necessary. Name and describe particular numerical methods required in the solution.