# Homework Assignment Number Four

**Problem (1)**

Consider the following boundary value problem

$$\frac{d^2 y}{dx^2} = c_1 \frac{dy}{dx} + c_2 y + c_3 \sin(x) + c_4 \exp\left(\frac{dy}{dx}\right)$$

with the boundary conditions

$$y(x=0) = y_o = 1.0$$
$$y(x=10) = y_f = 1.0$$

(a) Convert this single second-order ODE, to a system of two first-order ODEs.
(b) Let $c = [1,-2,2,0,]$ Determine the behavior of $y(x)$ and $y'(x)$ from $0 \le x \le 10$. Show the behavior in a plot. Clearly identify which curve corresponds to which function. State what value of the initial condition for $y'(x=0)$ led to the final solution.
(c) Let $c = [0,1,2,-2]$ Determine the behavior of $y(x)$ and $y'(x)$ from $0 \le x \le 10$. Show the behavior in a plot. Clearly identify which curve corresponds to which function. State what value of the initial condition for $y'(x=0)$ led to the final solution.

**Problem (2)**

Find an application from your own experience or a classical problem in your own field of research that results in a ODE boundary value problem.

(a) Describe the physical problem from which the equation arises. Describe it in sufficient detail that an engineer from a different discipline could understand it.
(b) Write the ODE(s). Write a complete set of reasonable boundary conditions.
(c) If known, provide the analytical solution.
(d) Numerically solve and plot the solution.
(e) Explain the physical significance of the solution(s) and its behavior.

(over)

**Problem (3)**

Consider the following one-dimensional linear parabolic PDE, commonly known as the heat equation,which describes the temperature, *T*, in the system as a function of axial position, *x*, and time, *t*,

$$\frac{\partial T}{\partial \mathrm{t}} = \alpha\left(\frac{\partial^2 T}{\partial x^2}\right) + \frac{h\mathrm{A}\left(\mathrm{T}_{\mathrm{surround}} - T(x,t)\right)}{\rho \hat{C}_p V}$$

where $\alpha$ , the thermal diffusivity is defined as the thermal conductivity, *k*, over the product of the density, $\rho$ , and specific heat capacity, $\hat{C}_p$ ,

$$\alpha = \frac{k}{\rho \hat{C}_p}$$

and where *A* is the surface area and *V* is the volume of the system, $\mathrm{T}_{\mathrm{surround}}$ is the temperature of the surroundings and *h* is an empirical heat transfer coefficient between the system and its surroundings.

Consider an aluminum cylindrical rod 1.0 meter long connecting two heat reservoirs. One of the reservoirs is maintained at T=300K, the other reservoir at T=400 K. Initially, the cylinder is at 300 K. There is no heat loss from the rod (*i.e.* *h*=0).

(i) Write the IC and BC’s.
(ii) What does the initial profile look like?
(iii) What does the steady state profile look like? Explain.
(iv) What is the temperature 0.5 meters into the rod at steady state?
(v) What is the temperature 0.5 meters into the rod after 1000 seconds?
(vi) Approximately how long does it take for the midpoint of the rod to get within 1% of the steady state value?

**Problem (4)**

Completely rework Problem (3) with the following initial and boundary conditions.
Consider an aluminum cylindrical rod 1.0 meter long with one end connected to a heat reservoir at T=400 K. The other end is insulated. The entire rod is also insulated so that there is no heat loss to the surroundings. The initial temperature of the rod is 300 K.

**Problem (5)**

Rework Problem (3) with the same initial conditions as given in Problem (3). However, in this case, the rod is not insulated so heat is lost from the rod, which is cylindrical and has a radius of 10 cm. The surrounding temperature is 200 K. Use a heat transfer coefficient of 40.0 W/m$^2$/K.