Homework Assignment Number One

For Problems 1 through 3 below, find
(a) the determinant of A
(b) the reduced row echelon form of A
(c) the reduced row echelon form of the Augmented Ab matrix.
(d) the rank of A.
(e) the rank of the augmented Ab matrix.
(f) the inverse of A if it exists
(g) a solution to $\underline{\underline{A}}\underline{x} = \underline{b}$ if it exists.

Problem 1.

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 5 & 1 \\ 1 & 4 & 2 \end{bmatrix} \text{ and } \underline{b} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

Problem 2.

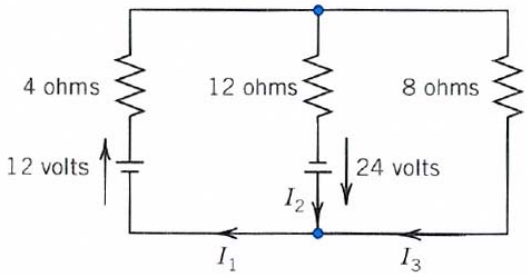
$$\underline{\underline{A}} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & -1 \\ 1 & 3 & 2 & -2 \\ 1 & 4 & 3 & -3 \end{bmatrix} \text{ and } \underline{b} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 2 \end{bmatrix}$$

Problem 3.

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 5 & 1 \\ 2 & 7 & 0 \end{bmatrix} \text{ and } \underline{b} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

Problem 4. (Kreyszig, 8th Edition, page 330, Problem Set 6.3, Problem 18)

Using Kirchoff's Current & Voltage laws, determine the three unknown currents.
(http://en.wikipedia.org/wiki/Kirchhoff's_circuit_laws)



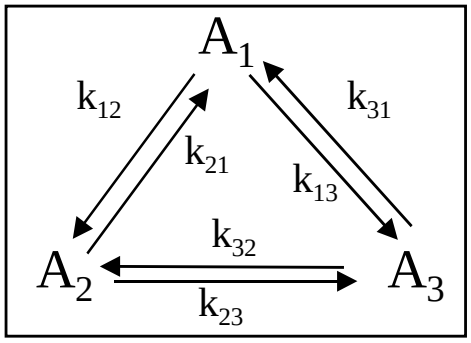
Problem 5.
Given the set of chemical reactions:

$$CH_4 + CH_4 \Leftrightarrow C_2H_6 + H_2$$
$$CH_4 + C_2H_6 \Leftrightarrow C_3H_8 + H_2$$
$$CH_4 + C_3H_8 \Leftrightarrow C_4H_{10} + H_2$$
$$C_2H_6 + C_2H_6 \Leftrightarrow C_4H_{10} + H_2$$
$$2CH_4 + 3O_2 \Leftrightarrow 2CO + 4H_2O$$
$$2C_2H_6 + 5O_2 \Leftrightarrow 4CO + 6H_2O$$
$$2C_3H_8 + 7O_2 \Leftrightarrow 6CO + 8H_2O$$
$$2C_4H_{10} + 9O_2 \Leftrightarrow 8CO + 10H_2O$$
$$2CO + O_2 \Leftrightarrow 2CO_2$$

(a) Write out the stoichiometric coefficient matrix.
(b) Determine the number of independent reactions using the stoichiometric coefficient matrix.
(c) Write a complete set of independent reactions.
(d) Write out the atomic matrix.
(e) Determine the number of independent reactions using the atomic matrix

Problem 6.
Find the eigenvalues and eigenvectors that describe the vibrational motion of acetylene, HCCH. Assume that the molecule is one-dimensional. Solve either symbolically in terms of m_H, m_c, k_{HC} and k_{CC}, or assign numbers to these four variables and solve numerically. Explain the significance of the eigenvalues and eigenvectors.

Problem 7.
Consider that you have a three-component reactive mixture, all undergoing reversible reactions, as pictured below:



In this picture, the A's are concentrations of the three species and the k's are rate constants. An example of this system is the kinetic equilibrium between para-, meta-, and ortho-xylene.

(a) Find the eigenvalues and eigenvectors for the following rate constants in a batch reactor.

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k12 = 0.50;
k21 = 0.25;
k13 = 0.20;
k31 = 0.05;
k23 = 0.30;
k32 = 0.15;
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(b) Give a physical interpretation of these eigenvalues and eigenvectors.
(c) Find the steady-state composition.