Final Examination
May 6, 2014

1. Multivariate Nonlinear Optimization

Download the data located at
http://utkstair.org/clausius/docs/mse506/data/mse506_xm02_p01.txt
The first column corresponds to wavelength. The second column corresponds to signal intensity. Perform a multivariate nonlinear optimization in order to fit this data to a series of Gaussian curves.

$$f(x;\mu,\sigma)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Determine the minimum number of Gaussian curves necessary and the mean and standard deviation of each.

2. Single Non-Linear Parabolic PDE

The one-dimensional heat equation can describe heat transfer in a material with both heat conduction and radiative heat loss.

$$\frac{\partial T}{\partial \mathrm{t}}=\frac{k}{\rho C_p}\frac{d^2T}{dz^2}-\frac{\varepsilon\sigma S}{\rho C_p}\left(T^4-T_s^{\,4}\right)$$

where the following variables [with units] are given as

- temperature in the material T [K]
- surrounding temperature $T_s = 300$ [K]
- axial position along material z [m]
- thermal conductivity $k = 401$ [J/K/m/s] (for Cu)
- mass density $\rho =$ 8960 [kg/m^3] (for Cu)
- heat capacity $C_p = 384.6$ [J/kg/K] (for Cu)
- Stefan–Boltzmann constant $\sigma = 5.670373x10^{-8}$ [J/s/m^2/K^4]
- gray body permittivity $\varepsilon = 0.15$ (for dull Cu)
- surface area to volume ratio $S = 200$ [m^{-1}] (for a cylindrical rod of diameter 0.05 m)

A cylindrical Cu rod of diameter 0.05 m and length 0.3 m is initially at $T(z,t=0)=1000$ K. One end of the rod is maintained at $T(z=0,t)=1000$ K. The other end of the rod is insulated,

$$\left.\frac{dT}{dz}\right|_{z=0.3}=0 \text{ K/m.}$$

(a) Plot the transient behavior.
(b) Find the approximate steady-state temperature in the material at z=0.3 m.

3. ODE Boundary Value Problem

Consider the following boundary value problem describing diffusion with reaction in a membrane.

$$0 = D\frac{d^2C_A}{dx^2} - kC_A$$

with the boundary conditions

$$C_A(x=0) = C_{A\,o} = 1.0 \text{ mol/m}^3$$
$$C_A(x=2) = C_{A\,f} = 0.1 \text{ mol/m}^3$$

where

$$D = 1.0 \cdot 10^{-5} \text{ m}^2\text{/s}$$
$$k = 1.0 \cdot 10^{-5} \text{ 1/s}$$

(a) Convert this single second-order ODE, to a system of two first-order ODEs.
(b) Plot the solution.
(c) What is the concentration gradient at x = 0?