Midterm Examination
February 25, 2014

1. Solution of a System of Nonlinear Algebraic Equations

Consider the set of nonlinear algebraic equations

$$0 = 10x_1 - 4x_2^3 + 9$$
$$0 = \exp(x_1) - 4x_2^2 + 1$$

(a) Use the multivariate Newton-Raphson method to find the roots to this system of equations near (2,2) and (-1,1). Report the RMS error on x and the number of iterations to converge.

2. Solution of a System of Nonlinear Ordinary Differential Equations

The Van der Pol oscillator is a non-conservative oscillator with non-linear damping. It evolves in time according to the second order differential equation,

$$\frac{d^2x}{dt^2} - \mu\left(1 - x^2\right)\frac{dx}{dt} + x = 0$$

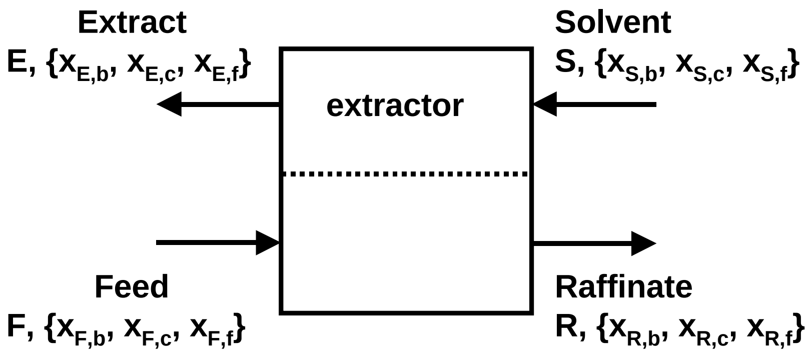
where *x* is the position coordinate, which is a function of the time *t*, and μ is a scalar parameter indicating the nonlinearity and the strength of the damping.

(a) Convert this second order ODE to a system of two first order ODEs.
(b) Use the classical fourth-order Runge Kutta method to solve this ODE from $t = 0$ to $t = 10$ for $\mu = 1/2$ and subject to the initial conditions, $x(t = 0) = 1$ and $x'(t = 0) = 0$. Sketch the plot and report the value of x at t = 10.

(Problem 3 on reverse side.)

3. Formulation and Solution of a System of Linear Algebraic Equations

Consider a liquid-liquid extractor as shown in the figure below that removes benzene from a primarily cyclohexane Feedstream using a furfural Solvent stream.



You are given all four flow rates and the compositions of the Feed (F) and Solvent (S) streams. Your task is to determine the composition of the two exiting streams, the Raffinate (R) and Extract (E).

$F = 100\, mol/hr$	$S = 150\, mol/hr$	$R = 95\, mol/hr$	$E = 155\, mol/hr$
$x_{F,b} = 0.1$	$x_{S,b} = 0.0010$	$x_{R,b} = ?$	$x_{E,b} = ?$
$x_{F,c} = 0.9$	$x_{S,c} = 0.0001$	$x_{R,c} = ?$	$x_{E,c} = ?$
$x_{F,f} = 0.0$	$x_{S,f} = 0.9989$	$x_{R,f} = ?$	$x_{E,f} = ?$

An analysis of the system yields six equations for your six unknowns.

(1) a benzene molar balance: $Rx_{R,b} + Ex_{E,b} = Fx_{F,b} + Sx_{S,b}$

(2) a cyclohexane molar balance: $Rx_{R,c} + Ex_{E,c} = Fx_{F,c} + Sx_{S,c}$

(3) raffinate mole fractions sum to unity: $x_{R,b} + x_{R,c} + x_{R,f} = 1$

(4) extract mole fractions sum to unity: $x_{E,b} + x_{E,c} + x_{E,f} = 1$

(5) benzene equilibrium constraint: $K_b = \frac{x_{E,b}}{x_{R,b}} = 20.0$

(6) c-hexane equilibrium constraint: $K_c = \frac{x_{E,c}}{x_{R,c}} = 0.01$

(a) Formulate the equations as a system of six linear algebraic equations in six unknowns.

(b) Convert the equations to matrix notation, $\underline{\underline{A}}\underline{x} = \underline{b}$. Identify $\underline{\underline{A}}$, $\underline{x}$ and $\underline{b}$.

(c) Determine and report the compositions of the Raffinate and Extract streams.