ChE 505 Final Exam Administered: Thursday, December 9, 1999

Problem (1)

Classify the following PDEs in terms of (a) linearity vs nonlinearity (b)hyperbolic, elliptic, or parabolic, and (c) spatial dimensionality.

$$\frac{\partial T}{\partial t} = -D\frac{\partial^2 T}{\partial x^2} + v\frac{\partial T}{\partial x} + k(T - T_o)$$
$$\frac{\partial^2 T}{\partial t^2} - a\frac{\partial T}{\partial t} = -c\frac{\partial^2 T}{\partial x^2} + k\sqrt{(T - T_o)}$$
$$0 = -c_x\frac{\partial^2 T}{\partial x^2} - c_y\frac{\partial^2 T}{\partial y^2} - c_z\frac{\partial^2 T}{\partial z^2} + kzT$$

Problem (2)

For each of the three PDEs in problem (1), give a complete set of initial conditions and boundary conditions.

Problem (3)

Describe in words and diagrams and algorithmic flowsheets, how one numerically accounts for a Neumann Boundary Condition of the type:

$$p(x,t)\frac{\partial T}{\partial x} = q(x,t)T + g(x,t)$$

Problem (4)

Classify the following order in terms of (i) linearity vs nonlinearity, (ii) Volterra vs Fredholm, (iii) IE of the first kind vs second kind.

$$\phi(\mathbf{x}) = f(\mathbf{x}) + \lambda \int_{0}^{\mathbf{x}} K(\mathbf{x}, \mathbf{s}) \phi(\mathbf{s}) d\mathbf{s}$$

Problem (5)

Numerically integrate using Trapezoidal or Simpson's 1/3, use intervals $n_y = n_x = 2$:

$$I = \int_{0}^{1} \int_{0}^{y} xy dx dy$$

Problem (6)

Consider water flowing through a horizontal pipe that is initially empty. If there is not enough flow to fill the pipe cross-section with water, set up the equations to describe the transient flow-system.