Exam Number One
Administered: Thursday, October 14, 1999

Problem (1)

Consider the system of nonlinear algebraic equations:

$$x_1^2 + x_2^2 + x_3^2 = 4$$
$$x_3 = x_1^2 + x_2^2$$
$$x_2 = \ln(x_1)$$

(a) How many solutions do you expect this equation to have?
(b) If we are going to solve this system using multivariate Newton-Raphson, we need the Jacobian and the residual. Determine them.
(c) For an initial guess of $(x_1, x_2, x_3) = (1,1,1)$, Evaluate the Jacobian and residual.
(d) Write the equation for the change in the unknowns and the new value of the unknowns at the next iteration.

Problem (2)

Say you are solving a system of nonlinear algebraic equations, using a numerical techniques, for example syseqn.m on MATLAB. You put in an initial guess for the unknowns and the program either crashes or doesn't converge. You are sure that there are no bugs in the program.

How do you proceed in solving the problem? What are you options? What are the possible sources of error?

Problem (3)

Consider the system of nonlinear ordinary differential equations:

$$\frac{dx_1}{dt} = x_1^2 + x_2^2 + x_3^2 - 4$$
$$\frac{dx_2}{dt} = x_1^2 + x_2^2 - x_3$$
$$\frac{dx_3}{dt} = \ln(x_1) - x_2$$

with the initial conditions $(x_1, x_2, x_3) = (1,1,1)$ at $t = 0$.
Perform one Euler step integration with a time step of $\Delta t = 0.1$.

Problem (4)

Say you are solving a system of nonlinear ODEs, using a numerical techniques, for example sysode.m on MATLAB. You enter the initial conditions and the program crashes. You are sure that there are no bugs in the program.

How do you proceed in solving the problem? What are you options? What are the possible sources of error?

Problem (5)

If the determinant of an nxn matrix $\underline{\underline{A}}$ is zero, what can you say about:

(a) the rank of the matrix
(b) the linear dependence of the equations which form the matrix
(c) the inverse of the matrix

(d) the eigenvalues of the matrix
(e) the eigenvectors of the matrix
(f) the number of solutions to $\underline{\underline{A}}\,\underline{x} = \underline{b}$

Problem (6)

If you are trying to solve a system of linear algebraic equations and it turns out that the determinant is zero, but you still want a solution, what do you do? Give the steps in the algorithm.