ChE/MSE 505
Midterm Examination
Administered: Monday, October 11, 2006

The general momentum balance,

$$\rho\frac{\partial \mathbf{v}}{\partial t} = -\rho\mathbf{v}\cdot\nabla(\mathbf{v}) - \nabla p - \nabla\cdot\boldsymbol{\tau} - \rho\nabla\hat{\Phi} \tag{1}$$

under the following assumptions: (1) incompressible flow, (2) one-dimensional system, (3) steady state, (4) external fields of $\nabla\hat{\Phi} = \frac{v - v_o}{\tau}$, (5) negligible pressure gradient, (6) Newtonian fluid, and (7) isothermal flow reduces to

$$0 = -\rho v\frac{\partial v}{\partial z} + \eta\frac{d^2 v}{dz^2} - \rho\frac{v - v_o}{\tau} \tag{2}$$

where v is velocity in the z direction, ρ is density, η is an elongational viscosity, and v_o is constant of the external field and τ is a strictly positive constant of the external field.

Answer the following questions and perform the following tasks.

1. Is equation (2) an ODE or PDE?
2. Is equation (2) linear or nonlinear?
3. Convert the second order DE equation in (2) to a system of first-order DEs.
4. Calculate the critical point of the system
5. Calculate the eigenvalues of this system of equations at the critical point.
6. Determine the type and stability of the critical point.