ChE/MSE 505
Fall, 2005
Final Examination

Consider a single component, inviscid fluid in a one dimensional system in the absence of any external fields (such as gravitational or electromagnetic). The material balance is given by

$$\frac{\partial \rho}{\partial t} = -\frac{\partial \rho v}{\partial x} \tag{1}$$

where ρ is the fluid density (kg/m^3), v is the velocity (m/s), x is the spatial coordinate (m), and t is the temporal coordinate (s). The momentum balance is given by

$$\rho \frac{\partial v}{\partial t} = -\rho v \frac{\partial v}{\partial x} - \frac{\partial p}{\partial x} \quad , \tag{2}$$

where p is the pressure (Pa). The energy balance is given by

$$\frac{\partial \rho \left(\frac{1}{2} v^2 + \hat{U} \right)}{\partial t} = -\frac{\partial}{\partial x} \rho \left(\frac{1}{2} v^3 + \hat{U} v \right) - \frac{\partial q}{\partial x} - \frac{\partial p v}{\partial x} \tag{3}$$

where $\hat{U}$ is the internal energy per mass (J/kg) and q is the conductive heat flux (J/m^2/s). In order to solve this system of three coupled, nonlinear, parabolic partial differential equations, we have to introduce some constitutive equations for p, $\hat{U}$, and q. For example, we might use Fourier's law, in which

$$q = -k_c \frac{\partial T}{\partial x} \tag{4}$$

where k_c is the thermal conductivity. For the heat capacity, one may use an approximate thermal equation of state,

$$\hat{U} = C_V T \tag{5}$$

For the pressure, one can use a mechanical equation of state, such as the ideal gas law, in which

$$p = \frac{\rho}{M} RT \tag{6}$$

where M is the molecular weight and R is the gas constant, which works when the fluid is a compressible gas. Another common assumption, used when the fluid is a liquid, is the assumption of incompressibility, in which the density is constant.

Problem 1.

Assume the fluid is incompressible. Solve/simplify as far as possible, then, if necessary, indicate what sort of numerical method is necessary to solve the problem.

Problem 2.

Continue Problem 1, assuming we are only interested in the steady state solution. Again, simplify as far as possible, if possible solve analytically, otherwise indicate the necessary numerical method needed to solve this problem.

Problem 3.

Comment on the stability of the steady state solution in Problem 2.

Problem 4.

Assume the fluid is an ideal gas. Solve/simplify as far as possible, then, if necessary, indicate what sort of numerical method is necessary to solve the problem.