ChE/MSE 505
Advanced Mathematic for Engineers
Final Exam
Fall Semester, 2004
Instructor: David Keffer
Administered: 8:00-10:00 am, Wednesday December 8, 2004

Problem 1.
Consider the partial differential equation describing the steady-state temperature profile in an uninsulated cylindrical metal rod of radius *R* and length *L*, in which there is variation in the radial (*r*) and axial (*z*) directions only. The two axial ends of the rod are held at fixed temperatures.

$$0 = \alpha \nabla^2 T \qquad (1)$$

where α is the thermal diffusivity. The Laplacian in two-dimensional cylindrical coordinates is defined as

$$\nabla^2 T = \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} . \qquad (2)$$

You may need the following properties: *h* is the heat transfer coefficient, ρ is the density, C_p is the heat capacity, k_c is the thermal conductivity, and T_{surr} is the temperature of the surroundings.

(a) Categorize the type of PDE: parabolic, hyperbolic, or elliptic.
(b) Determine the linearity of the PDE.
(c) Specify a complete set of consistent initial and boundary conditions for this problem.
(d) Identify and provide a detailed algorithm describing a numerical technique suitable for the solution of this problem.

Problem 2.
If the cylinder in problem 1 also loses heat to the surroundings via radiation, we must add an additional term to the energy balance in equation (1),

$$0 = \alpha \nabla^2 T - \frac{2\varepsilon\sigma}{R\rho C_p}\left(T^4 - T_{surr}^4 \right) \qquad (3)$$

where ε is the total emissivity of the cylinder and σ is a proportionality constant.
(e) Determine the linearity of the PDE.
(f) Identify and provide a detailed algorithm describing a numerical technique suitable for the solution of this problem.

Problem 3.
We want to use the following equation to fit some vapor pressure data.

$$P^{vap} = \exp\left(\frac{A}{B+T}\right) \tag{4}$$

where T is temperature and A and B are fitting constants. We have two pieces of data: the vapor pressure at 300 K is 1.1 atm and the vapor pressure at 320 K is 1.7 atm. Given this experimental data find the best values of A and B.