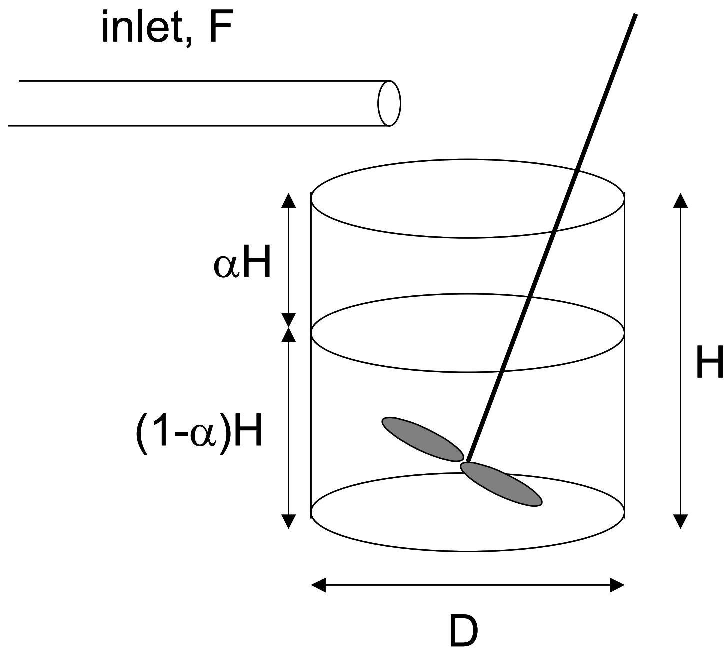
Midterm Examination
Administered: Thursday, December 5, 2002

Problem (1)
In the undergraduate unit operations laboratory at the University of TN, (ChE 310), the students model a continuous stirred tank reactor that is stirred only partially creating a two-cell model. The reactor is run in overflow mode, meaning that the effluent simply pours out over the top of the tank.



The tank is cylindrical with height, H, and diameter, D. The two-cell model assumes that the concentration is homogeneous in each cell, but different between cells. α is the volume fraction of the top cell. The inlet volumetric flow rate is F, with concentration C_{in}.

Consider a binary system of salt dissolved in water. (Thus there is actually no chemical reaction in the 'reactor'.) Writing mole balances on salt for each volume yields

accumulation = in – out

$$\tau_1 \frac{dC_1}{dt} = -C_1(t) + \left(\frac{R}{R+F}\right) C_2(t) + \left(\frac{F}{R+F}\right) C_{in}$$

$$\tau_2 \frac{dC_2}{dt} = C_1(t) - C_2(t)$$

where C_1 is the concentration of salt in the top cell and C_2 is the concentration of salt in the bottom cell, τ_1 and τ_2 are the residence times for each cell, given by:

$$\tau_1 = \left(\frac{\alpha V}{R+F}\right) \quad \text{and} \quad \tau_2 = \left(\frac{(1-\alpha)V}{R}\right) \quad \text{where} \quad V = \frac{H\pi D^2}{4}$$

R is the volumetric exchange rate between the top and bottom cells.
Answer the following questions. Assume that R, F, H, D, C_{in} and α are not functions of time.

(a) We have a system of two ordinary differential equations. Are they linear or nonlinear?
(b) Is the system of ODEs homogeneous or non-homogeneous?
(c) Does this system of ODEs have an analytical solution? If so, what is the **general** form?
(d) What initial conditions need to be known to define a unique solution to this problem?
(e) Analytically calculate and report the eigenvalues of this system of equations.
(f) What is the critical point (steady state) of the system?
(g) Knowing that R, F, H, D and α are all real numbers strictly greater than zero, characterize the type of critical point as a proper/improper node, saddle point, center, or spiral point. Determine the stability of the critical point.
(h) Is there any combination of R, F, H, D and α that can change the type or stability of the critical point?

Problem (2)

In problem (1), the two-cell model was used because a one-cell model failed to describe the experimental behavior of the poorly mixed reactor properly. Perhaps, the two-cell model described in problem (1) is also inadequate. One could increase the number of cells and create a three-cell model or a four-cell model, to allow finer gradations in the concentration as a function of axial position in the tank. Extending this idea, one could create a model with an infinite number of cells. In this model, each cell has the same fraction of volume, α, which is infinitesimally small.

(a) Write out the material balance(s) for the infinite-cell model.
(b) What kind of equation do you have: AE, ODE, PDE, or IE?
(c) Is the resulting equation linear or nonlinear?
(d) What initial and boundary conditions do you need?
(e) Describe in adequate detail the numerical algorithm that you would use to solve the problem. Discuss in particular any problems that you foresee.
(f) In the two-cell model, R is a single number with units of volumetric flowrate. Discuss the new form and units of R in the infinite-cell model.