ChE/MSE 505
Advanced Mathematic for Engineers
Final Exam
Fall Semester, 2001
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Consider the integro-differential equation

$$c_0(x)\frac{d\phi(x)}{dx}+c_1(x)\phi(x)+c_2(x)\left[\int_{x_o}^{x_f}N(x,y)\phi(y)dy\right]+c_3(x)=0$$

where

$$c_0(x)=1$$
$$c_1(x)=-1$$
$$c_2(x)=-1$$
$$c_3(x)=-e^{x}+1$$
$$N(x,y)=e^{x-y}$$
$$x_o=0$$
$$x_f=2$$

with the initial condition

$$\phi(x=x_o)=1$$

(a) Characterize the equation as linear or nonlinear.
(b) Use a numerical method to find an approximate solution to $\phi(x)$ from x_o to x_f. Use a discretization step of $\Delta x=1$. You are free to solve this as you choose, as long as you state your assumptions. However, I suggest you use a centered-finite difference formula to approximate the derivative at internal nodes and a backward-finite difference formula to approximate the derivative at the last node. Also, I suggest you use the trapezoidal rule to approximate the integral, although that too is not mandatory. I would like to see numerical values for the solution.