Review Handout for Statistics

μ = population mean, σ^2 = population variance, $\bar{x}$ = sample mean, s^2 = sample variance

Confidence Interval = $1-2\alpha$, in all cases, α represents area to left (less than probability)

A. Mean, population variance known

Use standard normal distribution

transformation: $Z = \dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}}$

symmetry: $z_{1-\alpha} = -z_\alpha$, where the area is to the left as in Table A.3 (Look up once.)

confidence interval: $P(\bar{X} + z_\alpha \dfrac{\sigma}{\sqrt{n}} < \mu < \bar{X} + z_{1-\alpha}\dfrac{\sigma}{\sqrt{n}}) = 1-2\alpha$

B. Mean, population variance unknown

Use t-distribution

transformation: $T = \dfrac{\bar{X}-\mu}{s/\sqrt{n}}$

symmetry: $t_{1-\alpha} = -t_\alpha$

parameters: $v = n-1$

confidence interval: $P(\bar{X} + t_\alpha \dfrac{s}{\sqrt{n}} < \mu < \bar{X} + t_{1-\alpha}\dfrac{s}{\sqrt{n}}) = 1-2\alpha$

C. Difference of Means, population variance known

Use standard normal distribution

transformation: $Z = \dfrac{\left(\bar{X}_1 - \bar{X}_2\right) - \left(\mu_1 - \mu_2\right)}{\sqrt{\left(\dfrac{\sigma_1^2}{n_1}\right) + \left(\dfrac{\sigma_2^2}{n_2}\right)}}$

symmetry: $z_{1-\alpha} = -z_\alpha$

confidence interval:

$$P\left[\left(\bar{X}_1 - \bar{X}_2\right) + z_\alpha\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \left(\mu_1 - \mu_2\right) < \left(\bar{X}_1 - \bar{X}_2\right) + z_{1-\alpha}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right] = 1-2\alpha$$

D. Difference of Means, population variance unknown

Use t-distribution

transformation: $T = \dfrac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\left(\dfrac{s_1^2}{n_1}\right) + \left(\dfrac{s_2^2}{n_2}\right)}}$

symmetry: $t_{1-\alpha} = -t_\alpha$

parameters: $v = n_1 + n_2 - 2$ if $\sigma_1 = \sigma_2$

parameters: $v = \dfrac{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)^2}{\left[\left(\dfrac{s_1^2}{n_1}\right)^2 \Big/ (n_1 - 1)\right] + \left[\left(\dfrac{s_2^2}{n_2}\right)^2 \Big/ (n_2 - 1)\right]}$ if $\sigma_1 \neq \sigma_2$

confidence interval:

$$P\left[(\overline{X}_1 - \overline{X}_2) + t_\alpha \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < (\mu_1 - \mu_2) < (\overline{X}_1 - \overline{X}_2) + t_{1-\alpha} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}\right] = 1 - 2\alpha$$

E. variance, population variance unknown

Use chi-squared distribution

transformation: $\chi^2 = \dfrac{(n-1)s^2}{\sigma^2}$

no symmetry

parameters: $v = n - 1$

confidence interval: $P\left[\dfrac{(n-1)s^2}{\chi^2_{1-\alpha}} < \sigma^2 < \dfrac{(n-1)s^2}{\chi^2_\alpha}\right] = 1 - 2\alpha$

F. ratio of variances, population variances unknown

Use F-distribution

transformation: $F = \dfrac{S_1^2 / \sigma_1^2}{S_2^2 / \sigma_2^2} = \dfrac{S_1^2 \sigma_2^2}{S_2^2 \sigma_1^2}$

parameters: $v_1 = n_1 - 1$, $v_2 = n_2 - 1$

symmetry: $\dfrac{1}{f_{1-\alpha}(v_1, v_2)} = f_\alpha(v_2, v_1)$

confidence interval: $P\left[\dfrac{S_1^2}{S_2^2} \dfrac{1}{f_{1-\alpha}(v_1, v_2)} < \dfrac{\sigma_1^2}{\sigma_2^2} < \dfrac{S_1^2}{S_2^2} \dfrac{1}{f_\alpha(v_1, v_2)}\right] = 1 - 2\alpha$