Exam II
Administered: Wednesday, February 28, 2001
28 points

For each problem part: 0 points if not attempted or no work shown,
1 point for partial credit, if work is shown,
2 points for correct numerical value of solution

Problem 1. (10 points)

We are developing a process where the quality of the feedstock is important. Poor quality feedstock can result in unacceptable product. A vendor for the feedstock provides us with 20 samples. He claims that the mean purity of the feed stock is 0.70 and claims that the standard deviation is 0.002. We run the 20 samples through our own lab and find a sample mean purity of 0.702 with a sample standard deviation of 0.003. Based on this information, answer the following questions.
(a) What PDF is appropriate for determining a confidence interval on the variance?
(b) Find the lower limit on a 96% confidence interval on the variance.
(c) Find the upper limit on a 96% confidence interval on the variance.
(d) Is the vendor's claim legitimate?
(e) If our maximum allowable standard deviation is 0.0045, can we be 96% confident that the vendor's feedstock is adequate?

Solution:

(a) What PDF is appropriate for determining a confidence interval on the variance?

Chi-squared distribution for the confidence interval on the variance.

(b) Find the lower limit on a 96% confidence interval on the variance.
(c) Find the upper limit on a 96% confidence interval on the variance.

$$\alpha = \frac{1 - C.I.}{2} = 0.02$$

$$v = n - 1 = 19$$

$$\chi^2_{\alpha} = 33.687$$ from table A.5

$$\chi^2_{1-\alpha} = 8.567$$ from table A.5

$$s^2 = 0.003^2 = 9 \cdot 10^{-6}$$

$$P\left[\frac{(n-1)s^2}{\chi^2_{\alpha}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\alpha}}\right] = 1 - 2\alpha$$

$$P\left[5.076 \cdot 10^{-6} < \sigma^2 < 19.960 \cdot 10^{-6}\right] = 0.98$$

(d) Is the vendor's claim legitimate?

$$\sigma^2 = 0.002^2 = 4 \cdot 10^{-6}$$

This variance does not fall within the confidence interval. The vendor's claim is not legitimate.

(e) If our maximum allowable standard deviation is 0.0045, can we be 96% confident that the vendor's feedstock is adequate?

$$\sigma_{max}{}^2 = 0.0045^2 = 20.25 \cdot 10^{-6}$$

Our entire interval falls below this maximum value. Therefore, we are at least 96% confident that the vendor's feedstock will meet our needs.

Problem 2. (8 points)

We decide to test our statistics savvy with one of the stars of the UT Lady Vols basketball team. Having compiled a shooting performance database on this particular player over the course of the season, we discover that her free throw shooting percentage is 0.782. In an on-the-spot demonstration, we ask the Lady Vol to attempt 10 free throws.

(a) What PDF would describe the probability that she make at least 9 of the 10 free throws?
(b) What is the probability that the Lady Vol make at least 9 of the 10 free throws?
(c) What PDF would describe the probability that she misses her first free throw on her last attempt?
(d) What is the probability that the Lady Vol misses her first free throw on her last attempt?

Solution:

(a) What PDF would describe the probability that she make at least 9 of the 10 free throws?

The binomial PDF, assuming the player is not streaky, and the probability of making a free throw is independent of whether she made the previous free throw.

(b) What is the probability that the Lady Vol make at least 9 of the 10 free throws?

X = number of successful free throws.

$$P(X = x) = b(x; n, p) = \binom{n}{x} p^x q^{n-x}$$
$$P(X \geq 9) = P(x = 9) + P(x = 10) = b(9; n, p) + b(10; n, p)$$
$$P(X \geq 9) = \binom{10}{9} 0.782^9 0.218^1 + \binom{10}{10} 0.782^{10} 0.218^0$$
$$P(X \geq 9) = 0.2384 + 0.0855 = 0.3239$$

(c) What PDF would describe the probability that she misses her first free throw on her last attempt?

The negative binomial PDF or the geometric PDF.

(d) What is the probability that the Lady Vol misses her first free throw on her last attempt?

here x = trial on which first failed free throw occurs, x=10.
p = probability of failed free throw = 0.218, q = 1- p

$$P(x = 10) = g(x = 10; p) = pq^{x\text{-}1} = 0.218 \cdot 0.782^9 = 0.0238$$

Problem 3. (6 points)

In studying dot-com businesses, we find that, on average, a company started in 1997, had an operational life of 2.7 years, before filing for bankruptcy, with a standard deviation of 1.4 years.

(a) What is the probability that a dot-com company can continue to operate for more than 5 years?
(b) How long does it take for 25% of the dot-com companies to fail?
(c) What PDF did you use to solve (a) & (b)?

Solution:

(a) What is the probability that a dot-com company can continue to operate for more than 5 years?

$$P(x > 5) = P(z > \frac{x-\mu}{\sigma}) = P(z > \frac{5-2.7}{1.4}) = P(z > 1.64) = 1 - P(z < 1.64)$$
$$P(x > 5) = 1 - 0.9495 = 0.0505$$

(b) How long does it take for 25% of the dot-com companies to fail?

$$P(z < z_{lo}) = 0.25$$
$$z_{lo} = -0.675 \quad \text{from table A.3}$$
$$z_{lo} = \frac{x_{lo} - \mu}{\sigma}$$
$$x_{lo} = \mu + \sigma z_{lo} = 2.7 + 1.4(-0.675) = 1.755 \text{ years}$$

(c) What PDF did you use to solve (a) & (b)?

The normal distribution.

Problem 4. (4 points)

The enthalpy of an ideal binary mixture can be approximated as the sum of the two pure component enthalpies, weighted by their respective mole fractions:

$$H_{mix} = x_1 H_1 + (1 - x_1) H_2 = x_1 C_{p,1}(T - T_{ref}) + (1 - x_1) C_{p,2}(T - T_{ref})$$

where x_1 is the mole fraction of component 1, $C_{p,1}$ is the heat capacity (assume it to be a constant) of component 1, T is the temperature, and T_{ref} is a reference temperature.

We have pure component heat capacities, empirically fit to data at the reference temperature, where our data yielded variances of $\sigma^2_{C_{p,1}}$ and $\sigma^2_{C_{p,2}}$.

What is the variance of the enthalpy of the mixture at composition x_1 and temperature T?

Solution:

What is the variance of the enthalpy of the mixture at composition x_1 and temperature T?

$$\sigma^2_{ax+by} = a^2 \sigma^2_x + b^2 \sigma^2_y + 2ab\sigma_{xy}$$

$$\sigma^2_{H_{mix}} = [x_1(T - T_{ref})]^2 \sigma^2_{C_{p,1}} + [(1 - x_1)(T - T_{ref})]^2 \sigma^2_{C_{p,2}} + 2[x_1(T - T_{ref})][(1 - x_1)(T - T_{ref})]\sigma_{C_{p,1}C_{p,2}}$$

You may assume that the two heat capacities are independent of each other. There is no physical reason not to believe this. If this is the case, then the covariance, $\sigma_{C_{p,1}C_{p,2}}$, is zero, so we have:

$$\sigma^2_{H_{mix}} = \left[x_1(T - T_{ref})\right]^2 \sigma^2_{C_{p,1}} + \left[(1 - x_1)(T - T_{ref})\right]^2 \sigma^2_{C_{p,2}}$$