Che 301 Exam I

Administered: Monday, September 20, 1999

20 points

For each problem part, 0 points if not attempted or no work shown,
1 point for partial credit, if work is shown,
2 points for correct numerical value of solution

Problem (1) (10 points total)

In the past few years, a chronic outbreak of deformities have been found among frogs living in ponds in Minnesota. The deformities result in misformed limbs, extra limbs, and missing limbs. In one study, it is found that for a frog population that is 59% female, the probability that a frog is deformed GIVEN that it is female is 0.0328. In the same frog population, the probability that a frog is normal GIVEN that it is male is 0.4516.

(a) Find the probability that a frog is deformed AND female. (2 points)
(b) Find the probability that a frog is male. (2 points)
(c) Find the probability that a frog is deformed GIVEN that it is male. (2 points)
(d) Find the probability that a frog is deformed AND male. (2 points)
(e) Find the probability that a frog is deformed. (2 points)

Solution:

(a) Find the probability that a frog is deformed AND female. (2 points)

$$P(D \cap F) = P(D | F)P(F) = 0.0328 \cdot 0.59 = 0.0194$$

(b) Find the probability that a frog is male. (2 points)

$$P(M) = 1 - P(F) = 1 - 0.59 = 0.41$$

(c) Find the probability that a frog is deformed GIVEN that it is male. (2 points)

$$P(D | M) = 1 - P(N | M) = 1 - 0.4516 = 0.5484$$

(d) Find the probability that a frog is deformed AND male. (2 points)

$$P(D \cap M) = P(D | M)P(M) = 0.5484 \cdot 0.41 = 0.2248$$

(e) Find the probability that a frog is deformed. (2 points)

$$P(D) = P(D \cap M) + P(D \cap F) = 0.2248 + 0.0194 = 0.2442$$

Problem (2) (4 points total)

(a) Determine if the following PDF is valid. (2 points)

$$f(x) = \begin{cases} \frac{3}{16}\left(x^2 - 2x\right) & 0 \le x \le 4 \\ 0 & otherwise \end{cases}$$

(b) For the following PDF, find $P\left(\frac{1}{2} < x < \frac{3}{2}\right)$ (2 points)

$$f(x) = \begin{cases} \frac{x}{2} & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

Solution:

(a) Although the function integrates to unity, it is not a valid PDF because it is not always greater than zero. From 0 to 2, f(x) < 0.

(b)

$$P\left(\frac{1}{2} < x < \frac{3}{2}\right) = \int_{1/2}^{3/2} f(x)dx = \int_{1/2}^{3/2} \frac{x}{2}dx = \left.\frac{x^2}{4}\right|_{1/2}^{3/2} = \frac{9}{16} - \frac{1}{16} = \frac{8}{16} = \frac{1}{2}$$

Problem (3) (6 points total)

Consider the PDF of problem 2(b) for 3(a) and 3(b).

(a) Find the mean of x. (2 points)

$$\mu_x = \int_0^2 xf(x)dx = \int_0^2 \frac{x^2}{2}dx = \left.\frac{x^3}{6}\right|_0^2 = \frac{8}{6} - 0 = \frac{4}{3}$$

(b) Find the mean of $(x - \mu_x)^2$. (2 points)

$$\mu_{h(x)} = \int_0^2 h(x)f(x)dx = \int_0^2 (x - \mu_x)^2 f(x)dx = \int_0^2 \left(x^2 - 2x\mu_x + \mu_x{}^2\right)f(x)dx$$

$$\mu_{h(x)} = \int_0^2 \left(x^2\right)f(x)dx + \int_0^2 \left(-2x\mu_x\right)f(x)dx + \int_0^2 \left(\mu_x{}^2\right)f(x)dx$$

$$\mu_{h(x)} = \int_0^2 \left(x^2\right)\frac{x}{2}dx + -2\mu_x\int_0^2 (x)f(x)dx + \mu_x{}^2\int_0^2 f(x)dx$$

$$\mu_{h(x)} = \left.\frac{x^4}{8}\right|_0^2 + -2\mu_x(\mu_x) + \mu_x{}^2(1) = 2 - \mu_x{}^2 = 2 - \left(\frac{4}{3}\right)^2 = \frac{18}{9} - \frac{16}{9} = \frac{2}{9}$$

(c) Consider the Joint PDF

$$f(x,y) = \begin{cases} \frac{x+y}{15} & 0 \le x \le 2, 0 \le y \le 3 \\ 0 & \text{otherwise} \end{cases}$$

Find the mean of xy. (2 points)

$$\mu_{h(x,y)} = \int_0^2\int_0^3 h(x,y)f(x,y)dydx = \int_0^2\int_0^3 xy\frac{x+y}{15}dydx = \frac{1}{15}\int_0^2\int_0^3 x^2y + xy^2 dydx$$

$$\mu_{h(x,y)} = \frac{1}{15}\int_0^2\left[\frac{x^2y^2}{2}+\frac{xy^3}{3}\right]_0^3 dx = \frac{1}{15}\int_0^2\frac{9x^2}{2}+9xdx = \frac{1}{15}\left[\frac{9x^3}{3}+\frac{9x^2}{2}\right]_0^2$$

$$\mu_{h(x,y)} = \frac{1}{15}\left[\frac{72}{6}+\frac{36}{2}\right] = \frac{12+18}{15} = \frac{30}{15} = 2$$