Final Exam
Administered: Monday, December 9, 2025
10:30 AM – 12:45 PM
30 points

Problem 1. (14 points)
The one-dimensional heat equation can describe heat transfer in silver with both heat conduction and radiative heat loss.

$$0 = \frac{d^2T}{dz^2} - \frac{\varepsilon\sigma S}{k_c}\left(T^4 - {T_s}^4\right)$$

A silver cylindrical rod has diameter 0.04 m and length 0.5 m. One end of the rod is maintained at $T(z = 0) = 600$ K. The other end of the rod is maintained at $T(z = L) = 800$ K. The variables and parameters are defined below.

- T is the temperature, K
- z is the axial position, m
- k_c is the thermal conductivity, $k_c = 429.0 \frac{W}{m \cdot K}$
- d is the diameter of the rod, $d = 0.04\ m$
- l is the length of the rod, $l = 0.5\ m$
- A is the surface area of the rod, $A = \pi dl$
- V is the volume of the rod, $V = \frac{\pi}{4} d^2 l$
- S is the surface area to volume ratio of the rod, $S = {}^{A}/_{V} = \frac{4}{d} = 100.0\ m^{-1}$
- T_{surr} is the surrounding temperature, $T_{surr} = 300\ K$
- σ is the Stefan–Boltzmann constant, $\sigma = 5.670373x10^{-8} \frac{W}{m^2 \cdot K^4}$
- ε is the gray body emissivity $\varepsilon = 0.04$

Answer the following questions and perform the following tasks.

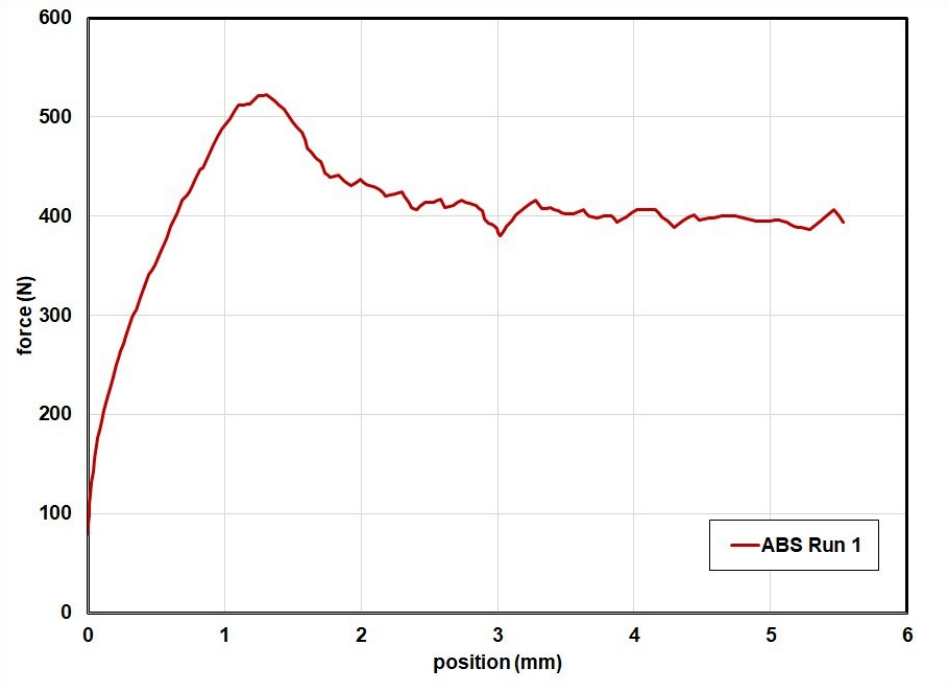
(a) Is this ODE problem linear or nonlinear?
(b) Is this ODE problem an initial value problem or a boundary value problem?
(c) Convert this second order ODE into a system of two first order ODEs.
(d) Find the initial temperature gradient at $z = 0$. Important hint: a good initial guess for the initial temperature gradient is the average value of the slope of the temperature.
(e) Sketch the temperature profile.
(f) Verify that your discretization resolution was sufficient.
(g) What is the temperature in the middle of the rod?

Problem 2. (6 points)

Work, W, is defined as the integral of the force, F, over distance, x,

$$W = \int_{x_o}^{x_f} F dx$$

The data for a tensile test experiment of Acrylonitrile Butadiene Styrene (ABS) is reported in the file "xm4p02_f25.txt" on the exam portion of the course website. The first column of the data is position in mm and the second column of the data is force in Newtons. A plot of the data is shown below.



(a) What method can you use to numerically integrate this data to obtain the work done during the tensile test?
(b) Report the numerical integral of this data.
(c) Convert this result to SI units, i.e. Joules.

Problem 3. (8 points)

Consider a set of three reactions occurring in a closed pot (a batch reactor) involving compounds, A, B and C.

number	reaction	rate expression	rate constant
1	A+A → B	$r_1 = k_1 A^2$	$k_1 = 7 \ \ l \cdot mol^{-1} s^{-1}$
2	B+A → C	$r_2 = k_2 A \cdot B$	$k_2 = 13 \ \ l \cdot mol^{-1} s^{-1}$
3	C → 3A	$r_3 = k_3 C$	$k_3 = 11 \ s^{-1}$

These equations give rise to the following steady state (at infinite time) mass balances.

compound	rate expression
A	$0 = 3k_3 C - 2k_1 A^2 - k_2 A \cdot B$
B	$0 = k_1 A^2 - k_2 A \cdot B$
C	$0 = k_2 A \cdot B - k_3 C$

We also recognize that the sum of the mass fractions equal unity.

$$A + B + C = 1$$

Your goal is to find the steady state composition in this reactor. To do so, answer the following questions.

(a) Are these equations linear or non-linear?
(b) Since you have three unknowns, which three of the four equations should be used to solve for the composition?
(c) What solution technique should use you to solve this problem?
(d) Provide a solution.

Problem 4. Project Question. (2 points)

Regarding the course project, answer the following question.

(a) In your opinion, what was the most significant result of your project?

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