

Exam III

Administered: Wednesday, November 5, 2025

20 points

For each problem part: 0 points if not attempted or no work shown,
1 point for partial credit, if work is shown,
2 points for correct numerical value of solution

Problem 1. (10 points)

In the elastic region, the stress-strain relationship is governed by Hooke's law in which the stress, σ , is related to the strain, ϵ , by elastic constants, c . Using Voigt notation, we can express this in three dimensions as

$$\underline{\sigma} = \underline{c}\underline{\epsilon}$$

where

$$\underline{\sigma} = \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{bmatrix} \quad \underline{\epsilon} = \begin{bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ 2\epsilon_{23} \\ 2\epsilon_{13} \\ 2\epsilon_{12} \end{bmatrix} \quad \underline{c} = \begin{bmatrix} c_{11} & c_{12} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{12} & c_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{44} \end{bmatrix}$$

where, for a material with a cubic crystal structure, there are only three independent non-zero elastic constants, c_{11} , c_{12} and c_{44} .

You are working with aluminum (FCC), which has values of these elastics given by

$$c_{11} = 104 \text{ GPa} \quad c_{12} = 51 \text{ GPa} \quad c_{44} = 159 \text{ GPa}$$

You perform an experiment where you measure the stresses in the materials to be

$$\underline{\sigma} = \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} 0.1907 \\ 0.1854 \\ 0.1801 \\ 0.0159 \\ 0.0477 \\ 0.0795 \end{bmatrix}$$

- Are these equations linear or non-linear?
- Rearrange the system of equations into the form of $\underline{Ax} = \underline{b}$. Identify each variable.
- Provide the determinant of the matrix of elastic constants.
- How many solutions are there to this system of equations?
- Determine the full set of strains within the material, ϵ_{11} , ϵ_{22} , ϵ_{33} , ϵ_{12} , ϵ_{13} and ϵ_{23} .

Problem 2. (10 points)

The file, http://utkstair.org/clausius/docs/mse301/data/xm3p02_f25.txt, contains results from a tensile test experiment. The tensile strain is given in column 1 and the tensile stress is given in column 2 with units of MPa.

- (a) Plot the stress vs strain curve.
- (b) Identify the region in which the elastic modulus can be extracted.
- (c) Perform the linear regression and report the mean value of the elastic modulus.
- (d) Also report the standard deviation of the elastic modulus.
- (e) Report the measure of fit of the model.