Exam I Solutions
Administered: Monday, September 16, 2024
24 points

For each problem part: 0 points if not attempted or no work shown,
1 point for partial credit, if work is shown,
2 points for correct numerical value of solution

Problem 1. (12 points)

Consider the data for the following **12** polymers given below. This data is available electronically on the course website in a spreadsheet file.

	specific gravity	tensile strength (psi)	tensile modulus of elasticity (psi)	tensile elongation (%)	coefficient of thermal expansion (in/in/°F x 10-5)
ABS	1.04	4100	294000	32	5.6
Acetal	1.42	10000	450000	75	6.8
Acrylic	1.19	10000	400000	4.5	4
High Impact Polystyrene	1.04	3500	270000	52	4.5
Nylon	1.14	12400	470000	90	4.5
PEEK	1.32	14000	490000	60	2.6
PET	1.38	11500	400000	70	3.9
PETG	1.27	7700	320000	210	3.8
PPS	1.35	12500	480000	4	4
PPSU	1.29	10100	340000	90	3.1
PVDF	1.78	7800	350000	35	7.1
Polycarbonate	1.2	9500	345000	135	3.8

Answer the following questions for the materials in this table.

(a) Determine the mean specific gravity.
(b) Determine the mean tensile strength.
(c) Determine the standard deviation of the specific gravity.
(d) Determine the standard deviation of the tensile strength.
(e) Determine the correlation coefficient between the specific gravity and tensile strength.
(f) What is the physical significance of your answer to part (e)?

Solution:
(a) Determine the mean specific gravity.

$$\mu_{\rho} = \frac{\sum_{i=1}^{n} \rho_i}{n} = 1.285$$

The mean specific gravity of these 12 materials is 1.285.

(b) Determine the mean tensile strength.

$$\mu_{\sigma_T} = \frac{\sum_{i=1}^{n} \sigma_{T_i}}{n} = 9425\ psi$$

The mean tensile strength of these 12 materials is 9425 psi.

(c) Determine the standard deviation of the specific gravity.

$$\sigma_\rho^2 = E[\rho^2] - E[\rho]^2 = 1.6875 - (1.285)^2 = 0.036275$$
$$\sigma_\rho = \sqrt{\sigma_\rho^2} = 0.19045997$$

The standard deviation of the specific gravity is 0.190

(d) Determine the standard deviation of the tensile strength.

$$\sigma_{\sigma_T}^2 = E[\sigma_T{}^2] - E[\sigma_T]^2 = 98309166.67 - (9425)^2 = 9478541.667(psi)^2$$

$$\sigma_{\sigma_T} = \sqrt{\sigma_{\sigma_T}^2} = 3078.724032\ psi$$

The standard deviation of the tensile strength is 3079 psi.

(e) Determine the correlation coefficient between the specific gravity and tensile strength.

First we determine the covariance.

$$\sigma_{\rho\cdot\sigma_T} = E[\rho\cdot\sigma_T] - E[\rho]E[\sigma_T] = 12288.08333 - 1.285\cdot 9425 = 176.9583333\ psi$$

With the covariance in hand, we can determine the correlation coefficient.

$$\rho_{\rho\cdot\sigma_T} = \frac{\sigma_{\rho\cdot\sigma_T}}{\sigma_\rho\cdot\sigma_{\sigma_T}} = \frac{176.9583333}{0.19045997\cdot 3078.724032} = 0.301784234$$

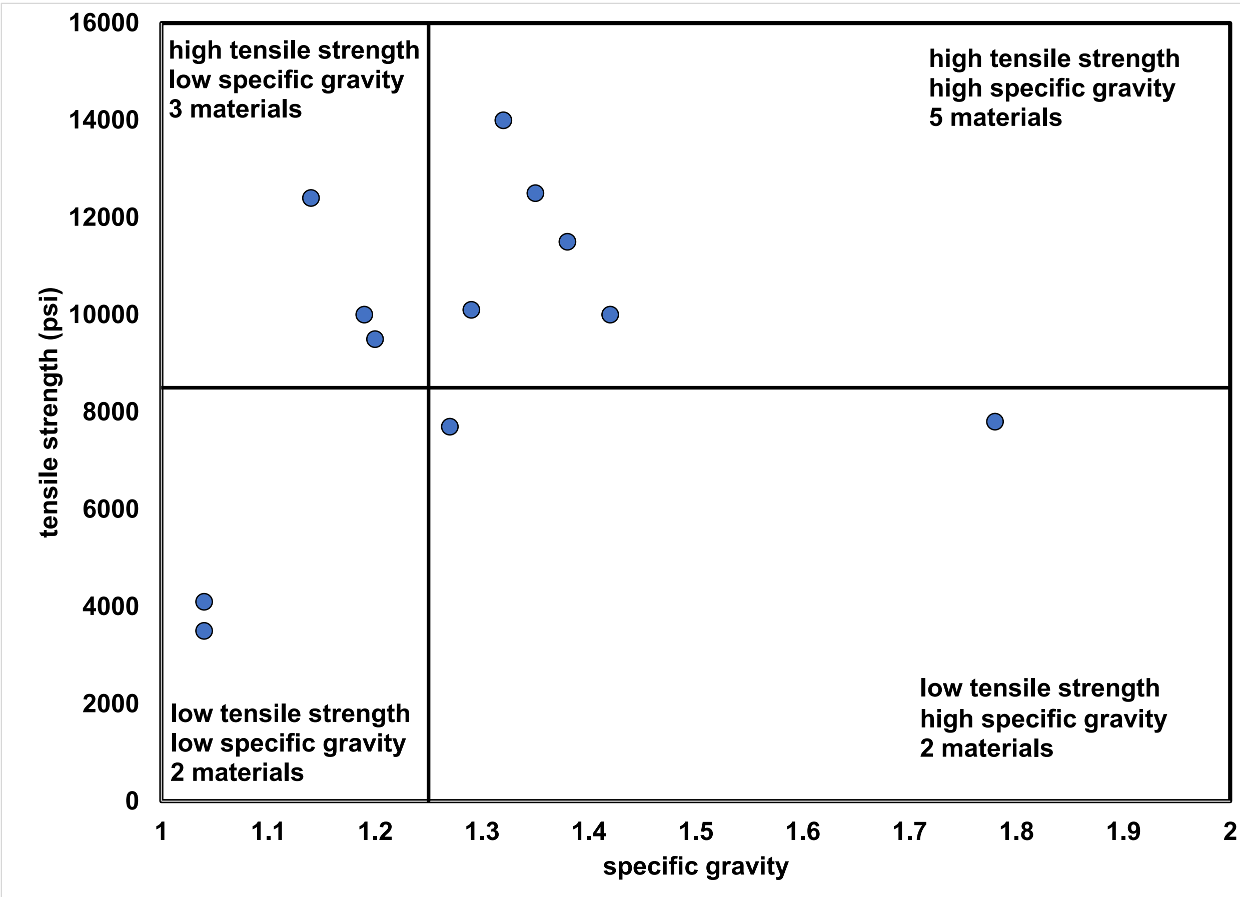
The correlation coefficient between the specific gravity and tensile strength is 0.302.

(f) What is the physical significance of your answer to part (e)?

The value of the correlation coefficient is positive but on the scale from 0 to 1, closer to 0 than 1. Therefore, there is a weak positive correlation between specific gravity and tensile strength. An increase in the specific gravity statistically corresponds to a modest increase in tensile strength.

Problem 2. (12 points)

Consider the **12** polymeric materials in the table in Problem 1. We are evaluating these materials in terms of low or high specific gravity and low or high tensile strength. A plot of the tensile strength vs the specific gravity is shown below.



Using this information, answer the following questions.

(a) Draw a Venn Diagram of the sample space for this data.
(b) What is the probability that a material has low tensile strength and high specific gravity?
(c) What is the probability that a material has high specific gravity?
(d) What is the probability that a material has low tensile strength given that it has high specific gravity?
(e) What is the probability that a material has high specific gravity given that it has low tensile strength?
(f) Given this classification, prove that tensile strength and specific gravity are not independent of each other.

Solution:

There are a total of 12 materials.
From the plot we see:

$$P(\sigma_{TS,lo} \cap \rho_{lo}) = \frac{2}{12}$$
$$P(\sigma_{TS,hi} \cap \rho_{lo}) = \frac{3}{12}$$
$$P(\sigma_{TS,lo} \cap \rho_{hi}) = \frac{2}{12}$$
$$P(\sigma_{TS,hi} \cap \rho_{hi}) = \frac{5}{12}$$

(a) Draw a Venn Diagram of the sample space for this experiment.

$\sigma_{TS,hi} \cap \rho_{lo}$	$\sigma_{TS,hi} \cap \rho_{hi}$
$\sigma_{TS,lo} \cap \rho_{lo}$	$\sigma_{TS,lo} \cap \rho_{hi}$

(b) What is the probability that a material has low tensile strength and high specific gravity?

From the data given,

$$P(\sigma_{TS,lo} \cap \rho_{hi}) = \frac{2}{12}$$

The probability that a material has low tensile strength and high specific gravity is $\frac{2}{12}$.

(c) What is the probability that a material has high specific gravity?

Consider the union probability rule.

$$P(\rho_{hi}) = P(\sigma_{TS,lo} \cap \rho_{hi}) + P(\sigma_{TS,hi} \cap \rho_{hi}) - P[(\sigma_{TS,lo} \cap \rho_{hi}) \cap (\sigma_{TS,hi} \cap \rho_{hi})]$$

$$= \frac{2}{12} + \frac{5}{12} - 0 = \frac{7}{12}$$

The probability that a material has a high specific gravity can be computed from the union rule. The intersection of low tensile strength and high tensile strength is zero by definition of the categorization of these materials. The probability that a material has a high hardness is $\frac{7}{12}$.

(d) What is the probability that a material has low tensile strength given that it has high specific gravity?

Consider the conditional probability rule.

$$P(\sigma_{TS,lo} | \rho_{hi}) = \frac{P(\sigma_{TS,lo} \cap \rho_{hi})}{P(\rho_{hi})} = \frac{2}{12} \Big/ \frac{7}{12} = \frac{2}{7}$$

The probability that a material has low tensile strength given that it has high specific gravity is $\frac{2}{7}$.

(e) What is the probability that a material has high specific gravity given that it has low tensile strength?

There are two equivalent ways to work this problem.
First, consider the conditional probability rule.

$$P(\rho_{hi} | \sigma_{TS,lo}) = \frac{P(\sigma_{TS,lo} \cap \rho_{hi})}{P(\sigma_{TS,lo})}$$

We can determine the denominator as

$$P(\sigma_{TS,lo}) = P(\sigma_{TS,lo} \cap \rho_{lo}) + P(\sigma_{TS,lo} \cap \rho_{hi}) - P[(\sigma_{TS,lo} \cap \rho_{lo}) \cap (\sigma_{TS,lo} \cap \rho_{hi})]$$

$$= \frac{2}{12} + \frac{2}{12} - 0 = \frac{4}{12}$$

$$P(\rho_{hi} | \sigma_{TS,lo}) = \frac{P(\sigma_{TS,lo} \cap \rho_{hi})}{P(\sigma_{TS,lo})} = \frac{2}{12} \Big/ \frac{4}{12} = \frac{1}{2}$$

The probability that a material has high specific gravity given that it has low tensile strength is $\frac{1}{2}$.

A second, alternate method to reach the same answer is to write the two conditional probabilities below and solve for the intersection

$$P(\sigma_{TS,lo}|\rho_{hi}) = \frac{P(\sigma_{TS,lo} \cap \rho_{hi})}{P(\rho_{hi})}$$

$$P(\rho_{hi}|\sigma_{TS,lo}) = \frac{P(\sigma_{TS,lo} \cap \rho_{hi})}{P(\sigma_{TS,lo})}$$

$$P(\sigma_{TS,lo} \cap \rho_{hi}) = P(\sigma_{TS,lo}|\rho_{hi})P(\rho_{hi}) = P(\rho_{hi}|\sigma_{TS,lo})P(\sigma_{TS,lo})$$

Solving the second equality for $P(\rho_{hi}|\sigma_{TS,lo})$ yields

$$P(\rho_{hi}|\sigma_{TS,lo}) = \frac{P(\sigma_{TS,lo}|\rho_{hi})P(\rho_{hi})}{P(\sigma_{TS,lo})} = \frac{\frac{2}{7}\frac{7}{12}}{\frac{4}{12}} = \frac{1}{2}$$

(f) Given this classification, prove that tensile strength and specific gravity are not independent of each other.

There are a variety of ways to prove this.

If A & B are independent, then $P(A|B) = P(A)$ or $P(B|A) = P(B)$.

So for example, we can show that hardness and density are not independent of each other through either of the following statements.

$$P(\rho_{hi}|\sigma_{TS,lo}) = \frac{1}{2} \neq P(\rho_{hi}) = \frac{7}{12}$$

$$P(\sigma_{TS,lo}|\rho_{hi}) = \frac{2}{7} \neq P(\sigma_{TS,lo}) = \frac{4}{12}$$

Since these statements are not equalities, we know that the variables are not independent of each other.