Final Exam

Administered: Tuesday, December 12, 2023
10:30 AM – 12:45 PM
32 points

Problem 1. (6 points)

Diffusion in solids is often an activated process, in which the diffusivity can be approximated by an Arrhenius-type temperature dependence.

$$D = D_o \, exp\left(-\frac{E_a}{RT}\right)$$

where D is the diffusivity in m^2/s, T is the temperature in K, R is the gas constant (8.314 J/mol/K), E_a is the activation energy in J/mol and D_o is the exponential prefactor in m^2/s.

(a) Linearize this equation so that it is linear in the unknown parameters, E_a and D_o.
(b) Using the table of data providing D and T in the file “xm4p01_f23.txt” on the exam portion of the course website, perform a linear regression to determine the mean values of E_a and D_o for the diffusion of Cu in solid Cu.
(c) Also report the standard deviations of E_a and D_o.

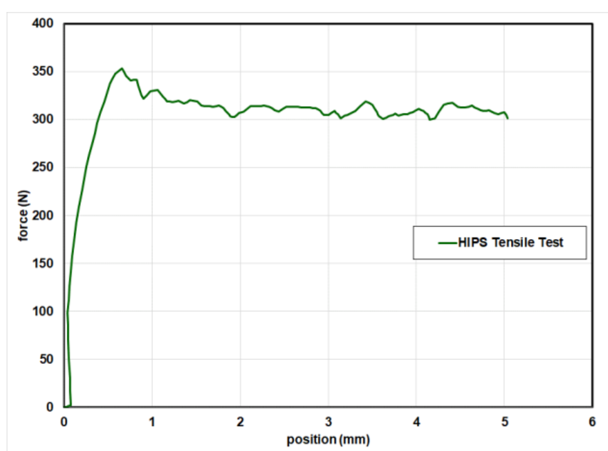
Problem 2. (6 points)

Work, W, is defined as the integral of the force, F, over distance, x,

$$W = \int_{x_o}^{x_f} F dx$$

The data for a tensile test experiment of High-Impact Polystyrene (HIPS) is reported in the file “xm4p02_f23.txt” on the exam portion of the course website. The first column of the data is position in mm and the second column of the data is force in Newtons. A plot of the data is shown below.

(a) What method can you use to numerically integrate this data to obtain the work done during the tensile test?
(b) Report the numerical integral of this data.
(c) Convert this result to SI units, i.e. Joules.



Problem 3. (14 points)
A cylindrical zinc rod, of diameter, d, and length L, is horizontally suspended between two heat reservoirs, which maintain the temperature at one end (z=0) at 300 K and at the other end (z=l) at 500 K. Between them a fan blows on the rod to conduct heat away. The steady state heat equation describing this set up is given below as

$$0 = \frac{k_c}{\rho C_p}\frac{d^2T}{dz^2} - \frac{h}{\rho C_p}\frac{A}{V}(T - T_{surr})$$

where

- k_c is the thermal conductivity, $k_c = 116.0 \frac{W}{m \cdot K}$
- ρ is the mass density, $\rho = 7140.0 \frac{kg}{m^3}$
- C_p is the specific heat capacity, $C_p = 387.0 \frac{J}{kg \cdot K}$
- d is the diameter of the rod, $d = 0.05\ m$
- l is the length of the rod, $l = 0.3\ m$
- A is the surface area of the rod, $A = \pi d l$
- V is the volume of the rod, $V = \frac{\pi}{4} d^2 l$
- A/V is the surface area to volume ratio of the rod, $A/V = \frac{4}{d}$
- T_{surr} is the surrounding temperature, $T_{surr} = 300\ K$
- h is an emprical heat transfer coefficient, $h = 40.0 \frac{W}{m^2 \cdot K}$

Answer the following questions and perform the following tasks.

(a) Is this ODE problem linear or nonlinear?
(b) Is this ODE problem an initial value problem or a boundary value problem?
(c) Convert this second order ODE into a system of two first order ODEs.
(d) Find the initial temperature gradient at z = 0.
(e) Sketch the temperature profile.
(f) Verify that your discretization resolution was sufficient.
(g) Report the temperature in the middle of the rod.

Problem 4. (6 points)

Answer the following questions based on the course project that you completed.

(a) Which project did you complete, “Phase Diagrams from Regular Solution Theory” or “Data Analysis of Carbon Fiber Tensile Tests”?
(b) Describe in your own words how the numerical method works in general for either “parameter stepping” or “outlier analysis”.
(c) Describe how either “parameter stepping” or “outlier analysis” was specifically applied in your project.