

Exam III

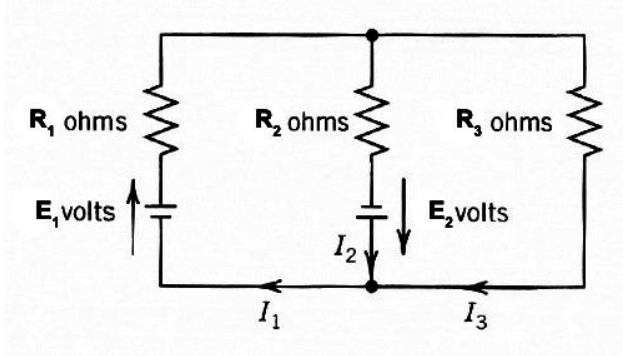
Administered: Wednesday, November 10, 2023

22 points

For each problem part: 0 points if not attempted or no work shown,  
1 point for partial credit, if work is shown,  
2 points for correct numerical value of solution

**Problem 1. (12 points)**

Consider the following circuit.



Circuit analysis is performed by using Kirchoff's Current & Voltage laws. Kirchoff's current law states that the sum of the currents entering and leaving any node must sum to zero. So for the bottom node, Kirchoff's current law provides the following equation.

$$-i_1 + i_2 + i_3 = 0$$

where  $i$  denotes current. Kirchoff's voltage law states that in a loop the sum of the voltages is zero. The application of Kirchoff's voltage law to the right and left loops respectively yields

$$R_1 i_1 + R_2 i_2 = E_1 + E_2$$

$$-R_2 i_2 + R_3 i_3 = -E_2$$

where  $R$  denotes a resistance and  $E$  denotes an applied voltage. Consider an example where the following values are assigned:  $R_1 = 40, R_2 = 30, R_3 = 20, E_1 = 60, E_2 = 40$

- Write this set of equations in matrix notation,  $\underline{A}\underline{x} = \underline{b}$ . Identify all three quantities,  $\underline{A}$ ,  $\underline{x}$  and  $\underline{b}$ .
- Calculate the determinant of  $\underline{A}$ .
- Calculate the rank of  $\underline{A}$ .
- Calculate the rank of  $\underline{A}|\underline{b}$
- How many solutions are there to this problem?
- Calculate the currents.

**Problem 2. (10 points)**

Consider the same circuit as was shown in problem one. However, this time the values of  $i_2$ ,  $R_2$  and  $E_1$  are unknown. We know the values of the other currents, resistances and applied voltages as follows:

$$i_1 = 2.0, i_3 = 1.0, R_1 = 40.0, R_3 = 30.0, E_2 = 20.0$$

- (a) Is this system of equation linear or nonlinear?
- (b) What numerical technique is appropriate for solving this problem?
- (c) Solve for the values of  $i_2$ ,  $R_2$  and  $E_1$ . (6 points)