Exam I
Administered: Tuesday, September 17, 2013
26 points

For each problem part: 0 points if not attempted or no work shown,
1 point for partial credit, if work is shown,
2 points for correct numerical value of solution

Problem 1. (10 points)

Consider the following mean values obtained for two discrete random variables, x and y.

variable	X	Y	X^2	Y^2	XY
mean	10.3500	5.5419	107.2074	30.8305	57.2907

(a) Find the variance of X.
(b) Find the standard deviation of X.
(c) Find the covariance of x and y.
(d) Find the correlation coefficient of x and y.
(e) Are x and y independent random variables?

Solution:

(a) Find the variance of X.

$$\sigma_X^2 = E\left[x^2\right] - E\left[x\right]^2 = 107.2074 - 10.35^2 = 0.0849$$

(b) Find the standard deviation of X.

$$\sigma_X = \sqrt{\sigma_X^2} = \sqrt{0.0849} = 0.2914$$

(c) Find the covariance of x and y.

$$\sigma_{XY} = E\left[xy\right] - E\left[x\right]E\left[y\right] = 57.2907 - 10.35 \cdot 5.5419 = -0.0680$$

(d) If the standard deviation of Y is 0.3433, find the correlation coefficient of x and y.

$$\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y} = \frac{-0.0068}{0.2914 \cdot 0.3433} = -0.6795$$

(e) Are x and y independent random variables?

No, x and y are not independent random variables because the covariance is not zero.

Problem 2. (8 points)

Consider the following PDF

$$f(x) = \begin{cases} c(x-1) & for\ 1 \le x \le 2 \\ 0 & otherwise \end{cases}$$

(a) Is this PDF continuous or discrete?
(b) Find the value of c that normalizes this PDF.
(c) Find the probability that x is between 1 and 3/2.
(d) Find the probability that x is greater than 3/2.

Solution:

(a) Is this PDF continuous or discrete?

This PDF is continuous.

(b) Find the value of c that normalizes this PDF.

$$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{\infty} c(x-1)dx = c\int_{1}^{2}(x-1)dx = c\left[\frac{x^2}{2} - x\right]_1^2 = c\left[0 - -\frac{1}{2}\right] = c\frac{1}{2} = 1$$

$$c = 2$$

(c) Find the probability that x is between 1 and 3/2.

$$P\left(1 < x < 3/2\right) = 2\int_{1}^{3/2}(x-1)dx = 2\left[\frac{x^2}{2} - x\right]_1^{3/2} = 2\left[-\frac{3}{8} - -\frac{1}{2}\right] = \frac{1}{4}$$

(d) Find the probability that x is greater than 3/2.

$$P\left(3/2 < x\right) = 1 - P\left(1 < x < 3/2\right) = 1 - \frac{1}{4} = \frac{3}{4}$$

Problem 3. (10 points)

Studies have shown that approximately 92% of the human population is right-handed (or right hand dominant). Recently, a study was performed to examine the relationship between handedness and location of linguistics ability in the human brain. The following results were published*.

	right-handed people	left-handed people
language dominant in right brain	5%	30%
language dominant in left brain	95%	70%

*McManus, I. C. 2002. Right Hand Left Hand. Great Britain: Weidenfeld & Nicolson, Ltd. 412p.

Using this information, answer the following questions.

(a) Draw a Venn Diagram of the sample space for the handedness and language dominance of a person.

(b) What is the probability that a person is language dominant in the left brain given that they are left handed?
(c) What is the probability that a person is language dominant in the right brain?
(d) What is the probability that a person is left-handed and language dominant in the left brain?

Solution:

We are given:

$$P(RH) = 0.92$$
$$P(RB \mid RH) = 0.05$$
$$P(LB \mid RH) = 0.95$$
$$P(RB \mid LH) = 0.30$$
$$P(LB \mid LH) = 0.70$$

(a) Draw a Venn Diagram of the sample space for the handedness and language dominance of a person.

$RH \cap RB$	$LH \cap RB$
$RH \cap LB$	$LH \cap LB$

(b) What is the probability that a person is language dominant in the left brain given that they are left handed?

This information was given in the problem statement.

$$P(LB \mid LH) = 0.70$$

(c) What is the probability that a person is language dominant in the right brain?

Consider the union rule.

$$P(RB) = P(RB \cap LH) + P(RB \cap RH)$$

$$P(RB) = P(RB \mid LH)P(LH) + P(RB \mid RH)P(RH)$$

$$P(LB) = 0.30 \cdot 0.08 + 0.05 \cdot 0.92 = 0.07$$

(d) What is the probability that a person is left-handed and language dominant in the left brain?

$$P(LH \cap LB) = P(LB \mid LH)P(LH) = 0.7 - \cdot 0.08 = 0.0560$$