Homework Assignment Number Seven
Solutions

On the course website, there are programs which solve:
(i) a single first-order linear ordinary integral equation
(ii) a single first-order non-linear ordinary integral equation

Problem 1.

Classify and numerically solve the following integral equation.

\[ \phi(x) = \frac{x^2}{100} + \frac{5}{2} \int_{5}^{x} e^{-\frac{(x+y)}{10}} \left[ \sin(\phi(y)) \right]^2 \, dy \]

Solve for \( x = 5 \) to 10.
Classify as linear/nonlinear, Volterra/Fredholm, first/second kind.
Provide a plot of the solution.
Demonstrate (1) the effect of changing the increment size (use say 5 and 20 intervals).
Demonstrate (2) the convergence of the method by looking at the solution at each iteration.

Solution:

This equation is a nonlinear Volterra equation of the second kind.
It is nonlinear in \( \phi \). It is Volterra because \( \phi \) appears both inside and outside the integral.
It is of the second kind because the limits of integration are variable.

Color code for following two plots:
red solid - solution after 1 iteration
blue solid - solution after 2 iterations
green solid - solution after 3 iterations
magenta solid - solution after 4 iterations
black dotted - solution after 5 iterations
red dotted - solution after 6 iterations
blue dotted - solution after 7 iterations
green dotted - solution after 8 iterations
We can see that the code is converging with each iteration. We can also see that we obtain a smoother solution with more intervals.