

Review Handout for Statistics

μ = population mean, σ^2 = population variance, \bar{X} = sample mean, S^2 = sample variance

Confidence Interval = $1 - 2\alpha$

A. Mean, population variance known

Use standard normal distribution

$$\text{transformation: } Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

symmetry: $z_{1-\alpha} = -z_{\alpha}$, where the area is to the left as in Table A.3 (Look up once.)

$$\text{confidence interval: } P\left(\bar{X} + z_{\alpha} \frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + z_{1-\alpha} \frac{\sigma}{\sqrt{n}}\right) = 1 - 2\alpha$$

B. Mean, population variance unknown

Use t-distribution

$$\text{transformation: } T = \frac{\bar{X} - \mu}{s / \sqrt{n}}$$

symmetry: $t_{1-\alpha} = -t_{\alpha}$ where the area is to the right as in Table A.4 (Look up once.)

parameters: $v = n - 1$

$$\text{confidence interval: } P\left(\bar{X} + t_{1-\alpha} \frac{s}{\sqrt{n}} < \mu < \bar{X} + t_{\alpha} \frac{s}{\sqrt{n}}\right) = 1 - 2\alpha$$

C. Difference of Means, population variance known

Use standard normal distribution

$$\text{transformation: } Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\left(\frac{\sigma_1^2}{n_1}\right) + \left(\frac{\sigma_2^2}{n_2}\right)}}$$

symmetry: $z_{1-\alpha} = -z_{\alpha}$, where the area is to the left as in Table A.3 (Look up once.)

confidence interval:

$$P\left[(\bar{X}_1 - \bar{X}_2) + z_{\alpha} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < (\mu_1 - \mu_2) < (\bar{X}_1 - \bar{X}_2) + z_{1-\alpha} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right] = 1 - 2\alpha$$

D. Difference of Means, population variance unknown

Use t-distribution

$$\text{transformation: } T = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\left(\frac{s_1^2}{n_1}\right) + \left(\frac{s_2^2}{n_2}\right)}}$$

symmetry: $t_{1-\alpha} = -t_\alpha$ where the area is to the right as in Table A.4 (Look up once.)

parameters: $v = n_1 + n_2 - 2$ if $\sigma_1 = \sigma_2$

$$\text{parameters: } v = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\left[\left(\frac{s_1^2}{n_1}\right)^2 / (n_1 - 1)\right] + \left[\left(\frac{s_2^2}{n_2}\right)^2 / (n_2 - 1)\right]} \text{ if } \sigma_1 \neq \sigma_2$$

confidence interval:

$$P\left[(\bar{X}_1 - \bar{X}_2) + t_{1-\alpha} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < (\mu_1 - \mu_2) < (\bar{X}_1 - \bar{X}_2) + t_\alpha \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}\right] = 1 - 2\alpha$$

E. variance, population variance unknown

Use chi-squared distribution

$$\text{transformation: } \chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

no symmetry, The area is to the right as in Table A.5 (Look up twice.)

parameters: $v = n - 1$

$$\text{confidence interval: } P\left[\frac{(n-1)s^2}{\chi_\alpha^2} < \sigma^2 < \frac{(n-1)s^2}{\chi_{1-\alpha}^2}\right] = 1 - 2\alpha$$

F. ratio of variances, population variances unknown

Use F-distribution

$$\text{transformation: } F = \frac{S_1^2 / \sigma_1^2}{S_2^2 / \sigma_2^2} = \frac{S_1^2 \sigma_2^2}{S_2^2 \sigma_1^2}$$

parameters: $v_1 = n_1 - 1, v_2 = n_2 - 1$

symmetry: $\frac{1}{f_{1-\alpha}(v_1, v_2)} = f_\alpha(v_2, v_1)$, The area is to the right as in Table A.6 (Look up twice.)

$$\text{confidence interval: } P\left[\frac{S_1^2}{S_2^2} \frac{1}{f_\alpha(v_1, v_2)} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{S_1^2}{S_2^2} f_\alpha(v_2, v_1)\right] = 1 - 2\alpha$$