**Midterm Examination Number Two**
**Administered: Wednesday, October 13, 1999**

In all relevant problems: WRITE DOWN THE FORMULA YOU USE, BEFORE YOU USE IT.
ALL PROBLEMS ARE WORTH 2 POINTS. EXAM HAS 26 POINTS.

**Problem (1)**

Consider flow down a circular pipe. The average velocity, $\bar{v}$, has a mean value of $\mu_{\bar{v}} = 0.131\frac{m}{s}$. The mean of the square of the average velocity is $\mu_{\bar{v}^2} = 0.482\frac{m}{s}$. The density, $\rho = 1000.0$ kg/m$^3$, the diameter, $D = 0.01$ m, and the viscosity, $\mu = 0.001\frac{kg}{m \cdot s}$ are constant.

(1.1) Find the standard deviation of the average velocity.

(1.2) Find the mean of the Reynolds number, $N_{Re} = \frac{D\bar{v}\rho}{\mu}$

(1.3) Is the flow laminar, transitional, or turbulent?

(1.4) Find the standard deviation of the Reynolds number.

(1.5) If we use the empirical correlation for the Nusselt Number: $N_{Nu} = 1.86\left(N_{Re}N_{Pr}\frac{D}{L}\right)^{\frac{1}{3}}$ where $N_{Pr}$ is the Prandtl number and $L$ is the length (both constants), would be expect the relative deviation of the Nusselt number (defined as $\frac{\sigma_{N_{Nu}}}{\mu_{N_{Nu}}}$) to be larger, the same as, or smaller than the relative deviation of the Reynolds number (defined as $\frac{\sigma_{N_{Re}}}{\mu_{N_{Re}}}$)? Why?

**Problem 2.**

We are in the business of manufacturing injection-molded plastic fenders to automobile makers. We claim that our fenders will remain intact under head-on impact with a standard concrete pylon up to an average speed of 24 mph with a standard deviation of 3 mph. Our competitor claims that they have developed a new additive to their plastic which allows their bumpers to remain intact under the same conditions up to an average speed of 30 mph with a standard deviation of 4 mph. We don't believe this claim one bit. We test 12 fenders, half with our fenders and half with the competition's fenders. From this sample, we find that our bumpers do not fracture until 24.2 mph with a standard deviation of 2.9 mph. From the competition's sample, we find that their bumpers do not fracture until 30.1 mph with a standard deviation of 10 mph.

(a) Find a 98% confidence interval for the difference in the two companies' product's average life-times, assuming the claimed population standard deviations are believable.

(b) Does the claimed average life-time difference fall within this confidence interval?

(c) Find a 98% confidence interval for the difference in the two companies' product's average life-times, assuming the claimed population standard deviations are not believable.

(d) Does the claimed average life-time difference fall within this confidence interval?

(e) Does the test in (c) allow for the possibility that our product has a higher mean than the competition?

**Problem 3.**

Before Christmas one year, we place out an advent wreath with four candles. Each candle has an average lifetime of 4 hours. If we light the candles at 9:00 p.m., what is the probability that at least 3 of them are still burning by midnight?

**Problem 4.**

Driving to school each morning, we encounter 6 traffic lights. Each traffic light stays green for 45 seconds, yellow for 5 seconds, and red for 50 seconds. Assuming that there is absolutely no synchronization among the streetlights and assuming that we don't run yellow lights, find the probability that in a single morning, we hit 2 green lights, 2 yellow lights, and 2 red lights.

**Problem 5.**

For the situation in problem (4), what is the average time spent waiting at a light on a single morning? Assume your arrival time is random (i.e. use the continuous uniform PDF).