Exam I
Administered: Monday, September 13, 2004
22 points

For each problem part: 0 points if not attempted or no work shown,
1 point for partial credit, if work is shown,
2 points for correct numerical value of solution

Problem 1. (16 points)

Consider the following PDF

$$f(x) = \begin{cases} c\left(x^2 - 1\right) & for\; 1 \le \text{x} \le 3 \\ 0 & otherwise \end{cases}$$

(a) Is this PDF continuous or discrete?
(b) Find the value of c that normalizes this PDF.
(c) Find the probability that x is between 1 and 2.
(d) Find the probability that x is greater than 2.
(e) Find the mean of the random variable x.
(f) Find the mean of the function of the random variable, $g(x) = 5x - 12$

Solution:

(a) Is this PDF continuous or discrete?

This PDF is continuous.

(b) Find the value of c that normalizes this PDF.

$$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{\infty} c\left(x^2 - 1\right)dx = c\int_{1}^{3}\left(x^2 - 1\right)dx = c\left[\frac{x^3}{3} - x\right]_1^3 = c\left[6 + \frac{2}{3}\right] = c\frac{20}{3} = 1$$

$$c = \frac{3}{20}$$

(c) Find the probability that x is between 1 and 2.

$$P(1 < x < 2) = \int_{1}^{2} f(x)dx = \frac{3}{20}\left[\frac{x^3}{3} - x\right]_1^2 = \frac{3}{20}\left[\frac{2}{3} + \frac{2}{3}\right] = \frac{1}{5}$$

(d) Find the probability that x is greater than 2.

$$P(x > 2) = 1 - P(1 < x < 2) = 1 - \frac{1}{5} = \frac{4}{5}$$

(e) Find the mean of the random variable x.

$$\mu_x = \int_{-\infty}^{\infty} x f(x) dx = \frac{3}{20}\int_1^3 x\left(x^2 - 1\right)dx = \frac{3}{20}\left[\frac{x^4}{4} - \frac{x^2}{2}\right]_1^3 = \frac{3}{20}\left[\frac{63}{4} + \frac{1}{4}\right] = \frac{12}{5}$$

(f) Find the mean of the function of the random variable, $g(x) = 5x - 12$.

Use the fact that the mean is a linear operator.

$$\mu_{g(x)} = 5\mu_x - 12 = 5\frac{12}{5} - 12 = 0$$

Problem 2. (10 points)

Studies have shown that approximately 92% of the human population is right-handed (or right hand dominant). Recently, a study was performed to examine the relationship between handedness and location of linguistics ability in the human brain. The following results were published*.

	right-handed people	left-handed people
language dominant in right brain	5%	30%
language dominant in left brain	95%	70%

*McManus, I. C. 2002. Right Hand Left Hand. Great Britain: Weidenfeld & Nicolson, Ltd. 412p.

Using this information, answer the following questions.

(a) Draw a Venn Diagram of the sample space for the handedness and language dominance of a person.
(b) What is the probability that a person is language dominant in the right brain given that they are right handed?
(c) What is the probability that a person is language dominant in the left brain?
(d) What is the probability that a person is right-handed and language dominant in the right brain?
(e) What is the probability that a person is right-handed, given that they are language dominant in the right brain?

Solution:

We are given:

$$P(RH) = 0.92$$
$$P(RB \mid RH) = 0.05$$
$$P(LB \mid RH) = 0.95$$
$$P(RB \mid LH) = 0.30$$
$$P(LB \mid LH) = 0.70$$

(a) Draw a Venn Diagram of the sample space for the handedness and language dominance of a person.

$RH \cap RB$	$LH \cap RB$
$RH \cap LB$	$LH \cap LB$

(b) What is the probability that a person is language dominant in the right brain given that they are right handed?

This information was given in the problem statement.

$$P(RB \mid RH) = 0.05$$

(c) What is the probability that a person is language dominant in the left brain?

Consider the union rule.

$$P(LB) = P(LB \cap LH) + P(LB \cap RH)$$

$$P(LB) = P(LB \mid LH)P(LH) + P(LB \mid RH)P(RH)$$

$$P(LB) = 0.70 \cdot 0.08 + 0.95 \cdot 0.92 = 0.9300$$

(d) What is the probability that a person is right-handed and language dominant in the right brain?

$$P(RH \cap RB) = P(RB \mid RH)P(RH) = 0.05 \cdot 0.92 = 0.0460$$

(e) What is the probability that a person is right-handed, given that they are language dominant in the right brain?

From problem (c) , we know that

$$P(RB) = 1 - P(LB) = 1 - 0.9300 = 0.07$$

$$P(RH \mid RB) = \frac{P(RH \cap RB)}{P(RB)} = \frac{0.0460}{0.07} = 0.6571$$