Cross-over in a 1-2 Heat Exchanger

Examination of the 1-2 heat exchanger. Crossover is possible only in the arrangement where the fluid in the tube is initially counter-current and finally co-current. (See attached diagram from Kern.)

Problem Specifications:

$T_{shell,in}$ = 400 K
$T_{tube,in}$ = 250 K
$\dot{m}_{tube}$ = 0.10 kg/s
$\dot{m}_{shell}$ = 0.20 kg/s
L = 6.0 m
$D_{tube,outside}$ = 0.0254 m
BWG = 10
N_{tube} = 24
D_{shell} = 0.25 m
K_{tube} = 45.0 W/m/K

Unknowns:

1) $T_{tube,out}$
2) $T_{shell,out}$
3) T_{inter}
4) $T_{crossover}$
5) T_{t1x}
6) α

Equations:

1) $q_1 = U_{o,1}A_{o,1}\Delta T_{lm,1} = \dot{m}_{tube}C_{p,1}\left(T_{inter} - T_{t1x}\right)$
2) $q_2 = U_{o,2}A_{o,2}\Delta T_{lm,2} = \dot{m}_{tube}C_{p,2}\left(T_{crossover} - T_{inter}\right)$
3) $q_3 = U_{o,3}A_{o,3}\Delta T_{lm,3} = \dot{m}_{tube}C_{p,3}\left(T_{tube,out} - T_{crossover}\right)$
4) $q_4 = U_{o,4}A_{o,4}\Delta T_{lm,4} = \dot{m}_{tube}C_{p,4}\left(T_{t1x} - T_{tube,in}\right)$
5) $q_5 = q_1 + q_2 = \dot{m}_{shell}C_{p,5}\left(T_{crossover} - T_{shell,in}\right)$
6) $q_7 = q_3 + q_4 = \dot{m}_{shell}C_{p,7}\left(T_{shell,out} - T_{crossover}\right)$

Solutions Requirements:

Solve a system of 6 highly non-linear algebraic equations simultaneously. This was done using MATLAB.

Tolerances for Convergence:

All temperatures within 0.01 K.
alpha within 0.001.

Initial Guesses for Six Unknowns:

$T_{tube,out}$ = 352.5 K
$T_{shell,out}$ = 350.0 K
T_{inter} = 301.25 K
$T_{crossover}$ = 355.0 K
T_{t1x} = 255.125 K
α = 0.9

Program Output:

SOLUTION CONVERGED!!!!
Section 1 Temp (K) inlet: 292.116079, outlet: 338.557987
Section 2 Temp (K) inlet: 338.557987, outlet: 364.458006
Section 3 Temp (K) inlet: 364.458006, outlet: 358.602875
Section 4 Temp (K) inlet: 250.000000, outlet: 292.116079
Section 5 Temp (K) inlet: 400.000000, outlet: 364.458006
Section 6 Temp (K) inlet: 400.000000, outlet: 364.458006
Section 7 Temp (K) inlet: 364.458006, outlet: 346.198057
Section 8 Temp (K) inlet: 364.458006, outlet: 346.198057
Section 1 heat transfer coefficients (W/m^2/K) h: 110.044836, Uo: 54.229114 Ui 74.054812
Section 2 heat transfer coefficients (W/m^2/K) h: 149.213424, Uo: 65.863988 Ui 89.943295
Section 3 heat transfer coefficients (W/m^2/K) h: 159.685980, Uo: 64.651455 Ui 88.287470
Section 4 heat transfer coefficients (W/m^2/K) h: 63.460598, Uo: 35.168477 Ui 48.025770
Section 1 heat transferred (W) q: -19427.295908, heat: 19429.548182
Section 2 heat transferred (W) q: -10860.580672, heat: 10894.243257
Section 3 heat transferred (W) q: 2455.379011, heat: -2471.723081
Section 4 heat transferred (W) q: -18025.216020, heat: 17835.177309
Section 5 heat transferred (W) q: -30287.876579, heat: -30230.337398
Section 6 heat transferred (W) q: -30287.876579, heat: -30230.337398
Section 7 heat transferred (W) q: -15569.837009, heat: -15382.401971
Section 8 heat transferred (W) q: -15569.837009, heat: -15382.401971
tube Overall heat transferred (W) q: -45857.713588, heat: 45687.245667
shell Overall heat transferred (W) q: -45857.713588, heat: -45612.739369
The position of the crossover point is 0.466603 percent of the length from the right edge.

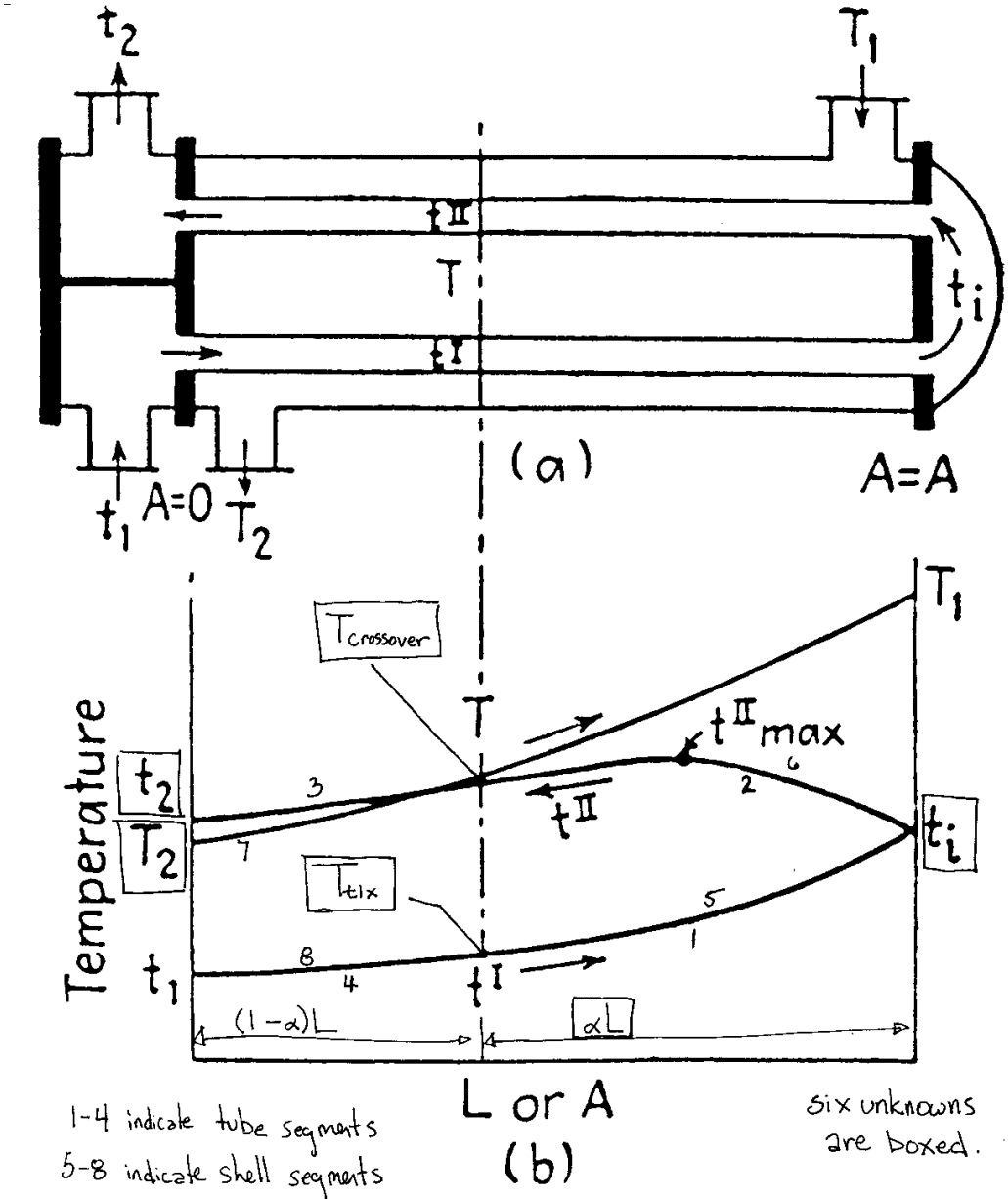

FIG. 7.21. Temperature relations in a 1-2 exchanger with conventional nozzle arrangement.

From: "Process Heat Transfer", Donald Q. Kern (Director, Process Engineering Division, The Patterson Foundry & Machine Company, and Adjunct Professor of Chemical Engineering, Polytechnic Institute of Brooklyn), First Edition, New York, McGraw-Hill Book Company, Inc.1950, pp. 140, 145.

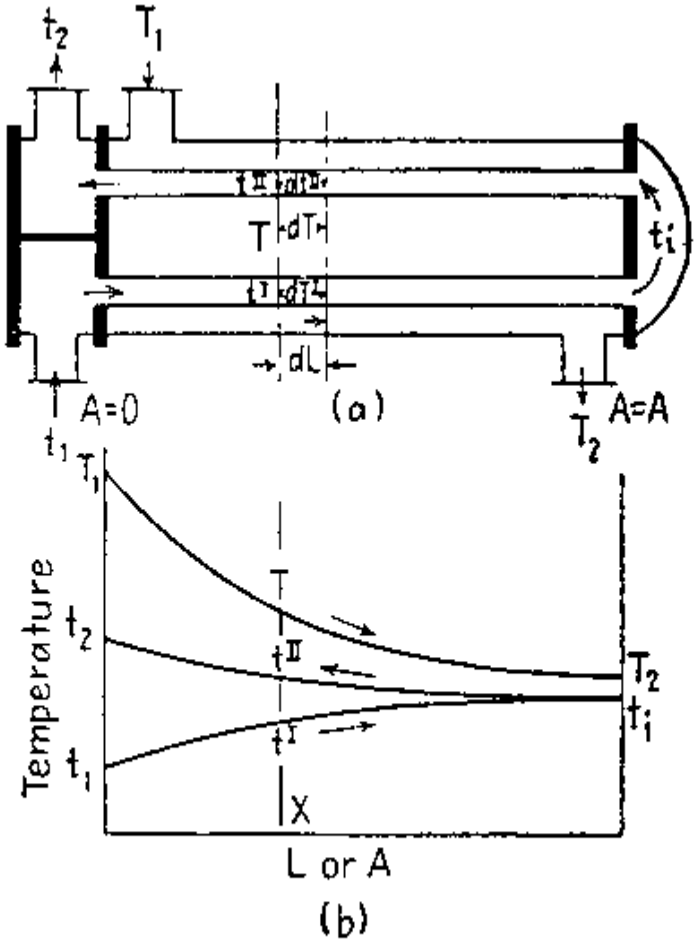

FIG. 7.20. Temperature relations in a 1-2 exchanger.

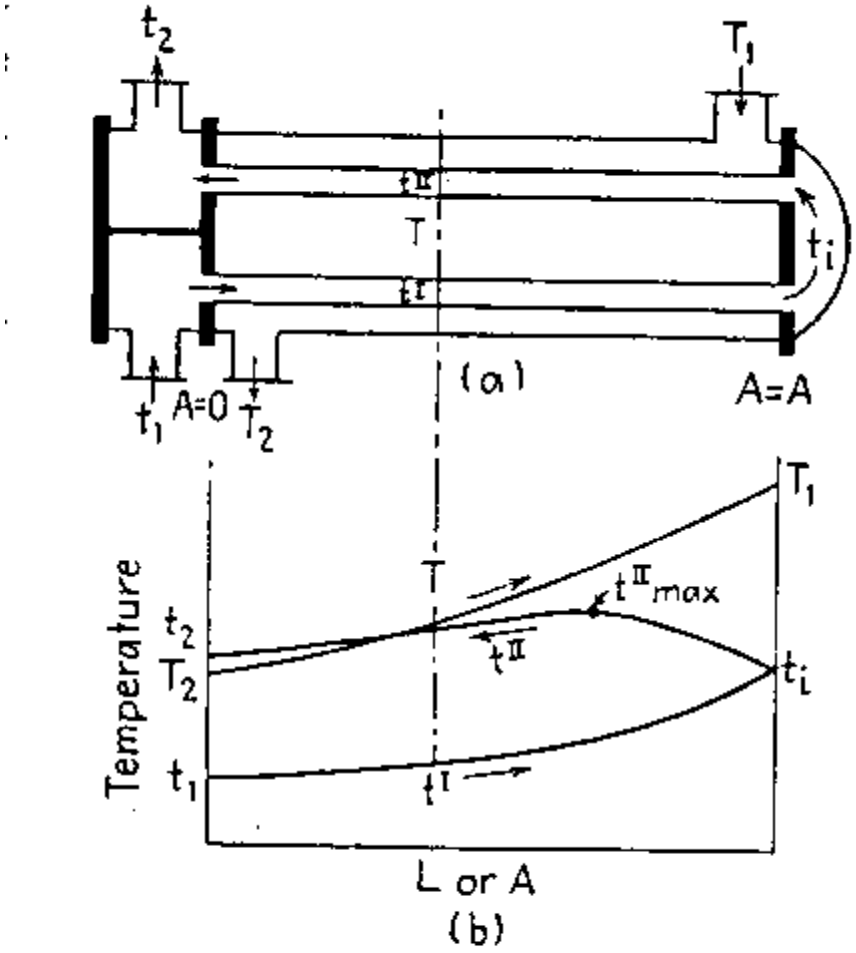

FIG. 7.21. Temperature relations in a 1-2 exchanger with conventional nozzle arrangement.