Homework Assignment Number Fourteen
Assigned: Wednesday, April 28, 1999
Due: Monday, May 3, 1999 BEFORE LECTURE STARTS.

Problem 1.

Consider the slab of finite thickness of $2H$ in the x-direction. Consider heat conduction without generation through this slab. The temperature is held constant on both sides of the plate. The initial temperature is constant through-out the plate. The properly posed problem is thus:

$$\frac{\partial T}{\partial t} = \alpha\left(\frac{\partial^2 T}{\partial x^2}\right), \qquad T(x,t=0) = T_o, \qquad T(x=0,t) = T_1, \qquad T(x=2H,t) = T_1$$

For this problem, use

$T_o = 300K$, $T_1 = 400K$, $H = 0.25m$, $\alpha = 9.8e-5\frac{m^2}{s}$

(a) Find the midpoint temperature at 200 seconds using the analytical solution

$$Y(x,t) = \frac{T_1 - T(x,t)}{T_1 - T_o} = \frac{4}{\pi}\sum_{n=1,3,5}^{\infty}\left[\frac{1}{n}\exp\left(\frac{-n^2\pi^2\alpha t}{4H^2}\right)\sin\left(\frac{n\pi x}{2H}\right)\right]$$

where x ranges from 0 to 2H in this equation. Specify what value of n, you used to approximate infinity.

(b) Find the midpoint temperature at 200 seconds using MATLAB.

(c) Compare (a) and (b).

Solution:

(a) T(x=0.25, t=200)=341.3097K.

The Fourier series converged to these 7 sig figs when n = 3.

(b)

for n = 200 time intervals and m = 20 space intervals, T(x=0.25, t=200)= 341.148 K

for n = 2000 time intervals and m = 20 space intervals, T(x=0.25, t=200)= 341.148 K

for n = 200 time intervals and m = 200 space intervals, T(x=0.25, t=200)= 341.116 K

(c)

Pretty close to the same number, as they should be.

Problem 2.

Consider the slab of finite thickness of $2H$ in the x-direction. Consider heat conduction without generation through this slab. The temperature is held constant on one side of the plate. The other side of the plate is insulated. The initial temperature is constant through-out the plate. There is an infinitely thin surface film over the side of the plate that is not insulated. The properly posed problem is thus:

$$\frac{\partial T}{\partial t} = \alpha\left(\frac{\partial^2 T}{\partial x^2}\right), \qquad T(x,t=0) = T_o, \qquad \frac{dT}{dx}(x=0,t) = 0, \quad T(x=2H,t) = T_1$$

For this problem, use

$T_o = 300K$, $T_1 = 400K$, $H = 0.0231m$, $k = 0.197\frac{W}{m\cdot K}$, $h = 8.52\frac{W}{m^2\cdot K}$, $\rho = 998\frac{kg}{m^3}$, $\hat{C}_p = 2300\frac{J}{kg\cdot K}$

(a) Find the midpoint temperature at 200 hours using the chart on page 341 of Geankoplis.

Note that this chart assume x is the distance from the center of the slab. Note also that in this chart, $x_1 = 2H$, which contradicts the picture on page 338, but which is correct. The picture is bad.

Solution:

(a)

$$Y(x,t) = \frac{T_1 - T(x,t)}{T_1 - T_o}$$

$$X(t) = \frac{\alpha t}{x_1^2} = \frac{8.5824e-008 \cdot 200 \cdot 3600}{0.0462^2} = 28.95$$

$$m = \frac{k}{hx_1} = 0.5$$

$$Y(x,t) = \frac{T_1 - T(x,t)}{T_1 - T_o} = 0.0$$ from plot on page 341

so (a) T(x=0.023, t=200 hours)=400K.