**Homework Assignment Number Five**
**Assigned: Wednesday, February 10, 1999**
**Due: Wednesday, February 17, 1999 BEFORE LECTURE STARTS.**

**Problem 1.** Geankoplis, problem 2.11-3, page 113

Air $T = 288K$, $p = 275000Pa$, isothermal compressible flow

$$D = 0.080m,\ L = 60m,\ MW = 29\frac{gram}{mol} = 0.029\frac{kg}{mol}$$

$$G = \frac{\dot{m}}{A} = 165.5\frac{kg}{m^2 \cdot s}$$

From appendix: $\mu = 1.8 \cdot 10^{-5}\frac{kg}{m \cdot s}$

$$\frac{\varepsilon}{D} = \frac{0.00015}{0.080} = 000575$$

$$N_{Re} = \frac{D\bar{v}\rho}{\mu} = \frac{DG}{\mu} = 7.35 \cdot 10^{-5}$$ therefore flow is turbulent

from table on page 88 of Geankoplis $f = 0.004$

From Equation (2.11-9) Geankoplis, page 102

$$p_1^{\ 2} - p_2^{\ 2} = \frac{4fLG^2RT}{D \cdot MW} + \frac{4G^2RT}{MW}\ln\left(\frac{p_1}{p_2}\right)$$

$$7.5625 \cdot 10^{10} - p_2^{\ 2} = 2.7139 \cdot 10^{10} + 4.5230 \cdot 10^9 \ln\left(\frac{275000}{p_2}\right)$$

Guess $p_2 = 200000$. Substitute in ln term on RHS. Solve for $p_2$ on LHS. Repeat, until guess is the same as result. Using, Excel to iteratively solve:

| iteration | pold | pnew |
|---|---|---|
| 1 | 200000 | 216900.1 |
| 2 | 216900.1 | 217744.2 |
| 3 | 217744.2 | 217784.5 |
| 4 | 217784.5 | 217786.5 |
| 5 | 217786.5 | 217786.6 |
| 6 | 217786.6 | 217786.6 |

So, $p_2 = 217786 \text{Pa}$ (2 pts)

$$v_{max} = \sqrt{\frac{RT}{MW}} = \sqrt{\frac{8.314 \cdot 288}{0.029}} = 287.3 \frac{m}{s}$$ (2 pts)

$$A = 0.0050 m^2 \qquad \dot{m} = GA = 0.832 \frac{kg}{s}$$

$$v_2 = \frac{\dot{m}}{A\rho_2} = \frac{\dot{m}}{A}\frac{RT}{p_2 MW} = 63 \frac{m}{s}$$ (2 pts)

**Problem 2.** Geankoplis, problem 3.1-4, page 205

water $T = 293K$, $p = 1atm$, $D = 1.0m$, $L = 10m$, $v = 1.2 \frac{m}{s}$

From appendix: $\mu = 1.00 \cdot 10^{-3} \frac{kg}{m \cdot s}$, $\rho = 998 \frac{kg}{m^3}$

$$N_{Re} = \frac{D_p \bar{v} \rho}{\mu} = \frac{(1)(1.2)(998)}{(1.00 \cdot 10^{-3})} = 1.192 \cdot 10^6$$ therefore flow is turbulent

and since $N_{Re} > 5 \cdot 10^5$, $C_D = 0.33$ (2 pts)

$$F_D = \left( C_D \frac{\rho \bar{v}^2}{2} \right) A_p = \left( 0.33 \frac{998.23 \cdot 1.2^2}{2} \right) (1 \cdot 10) = 2370N$$ (2 pts)

**Problem 3.** Geankoplis, problem 3.1-5, page 205

Packed bed of cubes: $L = 0.02\text{m}$, $\rho = 990\dfrac{\text{kg}}{\text{m}^3}$, $\rho_p = 1500\dfrac{\text{kg}}{\text{m}^3}$

The density of the bed is equal to the densities of void and solid weighted by their respective volume factions. The density of the void is zero.

$$\rho_{bed} = \varepsilon\rho_{void} + (1-\varepsilon)\rho_{solid}$$

$$\rho_{bed} = (1-\varepsilon)\rho_{solid}$$

$$\varepsilon = 1 - \frac{\rho_{bed}}{\rho_{solid}} = 1 - \frac{980}{1500} = 0.3467$$ (2pts)

The definition of the specific surface is the surface area over volume, which for a cube is;

$$a_v = \frac{S_p}{v_p} = \frac{6L^2}{L^3} = \frac{6}{L}$$

Now, Geankoplis gives us $\phi_s$ for a cube on the on page 122. If he had not, we can calculate $\phi_s$ by knowing that the sphericity is defined as the ratio of the diameter of a sphere to the length of the side of a cube, for a sphere and a cube with the same total volume.

$$V_{sph} = V_{cube}$$

$$\frac{4}{3}\pi\left(\frac{D_p}{2}\right)^3 = L^3$$

This gives an effective diameter of:

$$D_p = \sqrt[3]{\frac{6}{\pi}}L$$

The sphericity is the ratio of the surface area of the sphere to the surface area of the particle:

$$\phi_s = \frac{\pi D_p{}^2}{S_p} = \frac{\pi D_p{}^2}{6L^2} = \frac{\pi\left(\sqrt[3]{\frac{6}{\pi}}L\right)^2}{6L^2} = \left(\frac{\pi}{6}\right)^{1-2/3} = 0.806$$

Equating the two expressions for the specific surface , we find for the cube:

For our cube, $D_p = \sqrt[3]{\frac{6}{\pi}}L = 0.0248$ (2pts)

What this means is that a sphere with a diameter of $D_p = 0.0248$ has the same volume as a cube with sides of length $L = 0.02\text{m}$.

The ratio of surface area to volume is then:

$$a = a_v(1-\varepsilon) = \frac{6}{L}(1-\varepsilon) = 300(1-0.3467) = 196.0\frac{1}{\text{m}}$$

(2pts)

(b)

cylinders: $D = 0.02\text{m}$, $H = 0.03\text{m}$

The definition of the specific surface is the surface area over volume, which for a cylinder is;

$$a_v = \frac{S_p}{v_p} = \frac{H\pi D + 2\frac{\pi}{4}D^2}{H\frac{\pi}{4}D^2} = \frac{4HD + 2D^2}{HD^2}$$

$$a_v = \frac{4HD + 2D^2}{HD^2} = 266.67\frac{1}{\text{m}}$$

However Geankoplis does not give the sphericity for an arbitrary cylinder. One can obtain this by

$$V_{sph} = V_{cyl}$$

$$\frac{4}{3}\pi\left(\frac{D_p}{2}\right)^3 = H\pi\left(\frac{D}{2}\right)^2$$

Thus, the effective particle diameter of the cylinder is

$$D_p = \sqrt[3]{\frac{3}{2}HD^2}$$

The sphericity is the ratio of the surface area of the sphere to the surface area of the particle:

$$\phi_s = \frac{\pi D_p{}^2}{S_p} = \frac{\pi D_p{}^2}{H\pi D + 2\frac{\pi}{4}D^2} = \frac{\pi\left(\sqrt[3]{\frac{3}{2}HD^2}\right)^2}{H\pi D + 2\frac{\pi}{4}D^2} = \frac{4\left(\sqrt[3]{\frac{3}{2}HD^2}\right)^2}{4HD + 2D^2}$$

Now express H as a fraction of D, $H = kD$

$$\phi_s = \frac{4\left(\sqrt[3]{\frac{3}{2}kD^3}\right)^2}{4kD^2 + 2D^2} = \frac{2}{2k+1}\left(\frac{3}{2}k\right)^{2/3}$$

When H = D, k =1, and $\phi_s = \left(\frac{3}{2}\right)^{-1/3} = 0.874$ which is what Geankoplis gives.

For our values of the cylinder dimension:

$$\phi_s = \frac{4\left(\sqrt[3]{\frac{3}{2}HD^2}\right)^2}{4HD + 2D^2} = 0.8585$$

$$D_p = \sqrt[3]{\frac{3}{2}HD^2} = 0.0262$$ (2 pts)

What this means is that a sphere with a diameter of $D_p = 0.026$ has the same volume as a cylinder with diameter $D = 0.02m$ and height $H = 0.03m$

The ratio of surface area to volume is then:

$$a = a_v(1-\varepsilon) = 266.67(1-0.3467) = 174.2\frac{1}{m}$$ (2pts)

**Problem 4.** Geankoplis, problem 3.2-2, page 206

The fluid is air at 37.8 C = 311 K, D = 0.8 m, R = 0.124 m of water, static pressure is 0.275 m of water gage. The coefficient is 0.97

Calculate the density of air and the static pressure of the stack

$$\rho_{air} = \frac{p_{static} \cdot MW}{RT}$$
$$p_{static} = p_{atm} + gh(\rho_{water} - \rho_{air})$$

We have two equations and two unknowns, the density of air and the static pressure.   Substitute and solve.

$$p_{static} = p_{atm} + gh\left(\rho_{water} - \frac{p_{static} \cdot MW}{RT}\right)$$

$$p_{static} = \frac{p_{atm} + gh\rho_{water}}{\left(1 + \frac{ghMW}{RT}\right)} = 104017 \quad \text{Pa}$$ (2 pts)

$$\rho_{air} = \frac{p_{static} \cdot MW}{RT} = 1.167 \frac{kg}{m^3}$$ (2 pts)

$$v_{max} = C_p \sqrt{\frac{2(p_2 - p_1)}{\rho}} = 0.97\sqrt{\frac{2(9.8 \cdot 0.0124 \cdot (1000 - 1.167))}{1.167}}$$
$$v_{max} = 14.0 \frac{m}{s}$$ (2 pts)

$N_{Re} = \frac{D\bar{v}\rho}{\mu} = 690{,}000$ so flow is turbulent.  This tells us:

$$\bar{v} = 0.86 v_{max} = 12.04 \frac{m}{s}$$
$$q = \bar{v}A = 6.1 \frac{m^3}{s}$$

**Problem 5.**   Geankoplis, problem 3.2-5, page 207

The fluid is water at 20 C = 293 K, D = 0.0525 m, D2 = 0.020 m, R = 0.214 m of Hg.  The coefficient is 0.98

$$v_2 = \frac{C_v}{\sqrt{1-(D_2/D_1)^4}}\sqrt{\frac{2(p_2-p_1)}{\rho}}$$

$$= \frac{0.98}{\sqrt{1-(0.02/0.0525)^4}}\sqrt{\frac{2(9.8\cdot 0.214\cdot(13596-1000))}{1000}}$$ (2 pts)

$$= 0.99\cdot 7.27 = 7.20\frac{m}{s}$$

$$q = \bar{v}A = 2.3\cdot 10^{-3}\frac{m^3}{s}$$ (2 pts)

or

$$\dot{m} = \rho q = 2.3\frac{kg}{s}$$

**Problem 6.** Geankoplis, problem 3.2-6, page 207

The fluid is oil at 20 C = 293 K, density = 900 kg/m^3, viscosity = 6cp= 0.006 kg/m/s, D = 0.1023 m, q=0.0174 m^3/s, R = 0.93x10^5 Pa. The coefficient is 0.61

$$v_0 = \frac{q}{A} = \frac{4q}{\pi D_0^2}$$

$$v_0 = \frac{C_0}{\sqrt{1-(D_0/D_1)^4}}\sqrt{\frac{2(p_1-p_2)}{\rho}} = \frac{4q}{\pi D_0^2}$$

$$\frac{C_0^2}{1-(D_0/D_1)^4}\frac{2(p_1-p_2)}{\rho} = \left(\frac{4q}{\pi D_0^2}\right)^2$$

$$\left(\frac{\pi C_0}{4q}\right)^2\frac{2(p_1-p_2)}{\rho} = \left(\frac{1}{D_0^2}\right)^2\left[1-(D_0/D_1)^4\right]$$

$$\left(\frac{\pi C_0}{4q}\right)^2\frac{2(p_1-p_2)}{\rho} = \frac{1}{D_0^4} - \frac{1}{D_1^4}$$

$$D_0 = \sqrt[4]{\frac{1}{\left(\frac{\pi C_0}{4q}\right)^2 \frac{2(p_1 - p_2)}{\rho} + \frac{1}{D_1^4}}} = 0.0496\text{m}$$ (2 pts)

$$v_0 = \frac{q}{A} = \frac{4q}{\pi D_0^2} = 7.21\frac{\text{m}}{\text{s}}$$

$$v_1 = v_2 = \frac{q}{A} = \frac{4q}{\pi D_0^2} = 2.12\frac{\text{m}}{\text{s}}$$

$$N_{Re,0} = \frac{D_0 v_0 \rho}{\mu} = 53{,}600$$

$$N_{Re,1} = N_{Re,2} = \frac{D_1 v_1 \rho}{\mu} = 23{,}500$$

permanent pressure loss:

$$\frac{D_0}{D_1} = 0.48$$

So, from Geankoplis, page 132,

$$\Delta p_{perm} = 0.73(p_2 - p_1) = 67890\text{Pa}$$