Problem 1. Geankoplis, problem 2.11-3, page 113

Air \( T = 288K, p = 275000 \text{Pa} \), isothermal compressible flow

\[
D = 0.080 \text{m}, L = 60 \text{m}, MW = 29 \text{gram/mol} = 0.029 \text{ kg/mol}
\]

\[
G = \frac{\dot{m}}{A} = 165.5 \frac{\text{kg}}{\text{m}^2 \cdot \text{s}}
\]

From appendix: \( \mu = 1.8 \times 10^{-5} \frac{\text{kg}}{\text{m} \cdot \text{s}} \)

\[
\frac{\varepsilon}{D} = \frac{0.00015}{0.080} = 0.0001875
\]

\[
N_{Re} = \frac{D \dot{V} \rho}{\mu} = \frac{DG}{\mu} = 7.35 \times 10^{-5} \text{ therefore flow is turbulent}
\]

from table on page 88 of Geankoplis \( f = 0.004 \)

From Equation (2.11-9) Geankoplis, page 102

\[
\begin{align*}
\rho_1^2 - \rho_2^2 &= \frac{4fL G^2 RT}{D \cdot MW} + \frac{4G^2 RT}{MW} \ln \left( \frac{\rho_1}{\rho_2} \right) \\
7.5625 \times 10^{10} - \rho_2^2 &= 2.7139 \times 10^{10} + 4.5230 \times 10^9 \ln \left( \frac{275000}{\rho_2} \right)
\end{align*}
\]

Guess \( \rho_2 = 200000 \). Substitute in ln term on RHS. Solve for \( \rho_2 \) on LHS. Repeat, until guess is the same as result. Using, Excel to iteratively solve:
iteration | pold   | pnew   \\
---|--------|--------\
1  | 200000 | 216900.1 \\
2  | 216900.1 | 217744.2 \\
3  | 217744.2 | 217784.5 \\
4  | 217784.5 | 217786.5 \\
5  | 217786.5 | 217786.6 \\
6  | 217786.6 | 217786.6 \\

So, \( p_2 = 217786 \text{Pa} \)  

\[
\nu_{\text{max}} = \sqrt{\frac{RT}{MW}} = \sqrt{\frac{8.314 \cdot 288}{0.029}} = 287.3 \text{ m/s} 
\]

\( A = 0.0050 \text{m}^2 \) \( \dot{m} = GA = 0.832 \frac{\text{kg}}{\text{s}} \)

\[
\nu_2 = \frac{\dot{m}}{A\rho_2} = \frac{\dot{m} RT}{A p_2 MW} = 63 \text{ m/s} 
\]

**Problem 2.** Geankoplis, problem 3.1-4, page 205

water \( T = 293 \text{K}, p = 1 \text{atm}, D = 1.0 \text{m}, L = 10 \text{m}, \nu = 1.2 \frac{\text{m}}{\text{s}} \)

From appendix: \( \mu = 1.00 \cdot 10^{-3} \frac{\text{kg}}{\text{m} \cdot \text{s}}, \rho = 998 \frac{\text{kg}}{\text{m}^3} \)

\[
N_{\text{Re}} = \frac{D_p \nu \rho}{\mu} = \frac{(1)(1.2)(998)}{(1.00 \cdot 10^{-3})} = 1.192 \cdot 10^6 \text{ therefore flow is turbulent} 
\]

and since \( N_{\text{Re}} > 5 \cdot 10^5 \), \( C_D = 0.33 \)

\[
F_D = \left( C_D \frac{\rho \nu^2}{2} \right) A_p = \left( 0.33 \frac{998.23 \cdot 1.2^2}{2} \right) (1 \cdot 10) = 2370 \text{N} 
\]

**Problem 3.** Geankoplis, problem 3.1-5, page 205
Packed bed of cubes: \( L = 0.02 \text{m} \), \( \rho = 990 \frac{\text{kg}}{\text{m}^3} \), \( \rho_p = 1500 \frac{\text{kg}}{\text{m}^3} \)

The density of the bed is equal to the densities of void and solid weighted by their respective volume fractions. The density of the void is zero.

\[
\rho_{\text{bed}} = \varepsilon \rho_{\text{void}} + (1 - \varepsilon) \rho_{\text{solid}}
\]

\[
\rho_{\text{bed}} = (1 - \varepsilon) \rho_{\text{solid}}
\]

\[
\varepsilon = 1 - \frac{\rho_{\text{bed}}}{\rho_{\text{solid}}} = 1 - \frac{980}{1500} = 0.3467
\]

The definition of the specific surface is the surface area over volume, which for a cube is:

\[
a_v = \frac{S_p}{V_p} = \frac{6L^2}{L^3} = \frac{6}{L}
\]

Now, Geankoplis gives us \( \phi_s \) for a cube on the on page 122. If he had not, we can calculate \( \phi_s \) by knowing that the sphericity is defined as the ratio of the diameter of a sphere to the length of the side of a cube, for a sphere and a cube with the same total volume.

\[
V_{\text{sph}} = V_{\text{cube}}
\]

\[
\frac{4}{3} \pi \left( \frac{D_p}{2} \right)^3 = L^3
\]

This gives an effective diameter of:

\[
D_p = \frac{\sqrt[3]{6L}}{\pi}
\]

The sphericity is the ratio of the surface area of the sphere to the surface area of the particle:

\[
\phi_s = \frac{\pi D_p^2}{S_p} = \frac{\pi D_p^2}{6L^2} = \frac{\pi \left( \frac{\sqrt[3]{6L}}{\pi} \right)^2}{6L^2} = \left( \frac{\pi}{6} \right)^{1-2/3} = 0.806
\]

Equating the two expressions for the specific surface, we find for the cube:
For our cube, $D_p = \frac{3\sqrt[3]{6}}{\pi} L = 0.0248$ 

What this means is that a sphere with a diameter of $D_p = 0.0248$ has the same volume as a cube with sides of length $L = 0.02 \text{ m}$.

The ratio of surface area to volume is then:

$$a = a_v (1 - \varepsilon) = \frac{6}{L} (1 - \varepsilon) = 300 (1 - 0.3467) = 196.0 \frac{1}{\text{m}}$$

(b) cylinders: $D = 0.02 \text{ m}, \ H = 0.03 \text{ m}$

The definition of the specific surface is the surface area over volume, which for a cylinder is:

$$a_v = \frac{S_p}{V_p} = \frac{H \pi D + 2 \pi D^2}{H \frac{\pi D^2}{4}} = \frac{4HD + 2D^2}{HD^2}$$

$$a_v = \frac{4HD + 2D^2}{HD^2} = 266.67 \frac{1}{\text{m}}$$

However Geankoplis does not give the sphericity for an arbitrary cylinder. One can obtain this by

$$V_{sph} = V_{cyl}$$

$$\frac{4}{3} \pi \left(\frac{D_p}{2}\right)^3 = H \pi \left(\frac{D}{2}\right)^2$$

Thus, the effective particle diameter of the cylinder is

$$D_p = \frac{3}{\sqrt{2}} HD^2$$

The sphericity is the ratio of the surface area of the sphere to the surface area of the particle:
\[
\phi_s = \frac{\pi D_p^2}{S_p} = \frac{\pi D_p^2}{H \pi D + 2 \frac{\pi}{4} D^2} = \frac{\pi \left( \frac{3}{\sqrt{2}} \frac{H D^2}{4} \right)^2}{H \pi D + 2 \frac{\pi}{4} D^2} = \frac{4 \left( \frac{3}{\sqrt{2}} \frac{H D^2}{4} \right)^2}{4H \pi D + 2D^2}
\]

Now express \( H \) as a fraction of \( D \), \( H = kD \)

\[
\phi_s = \frac{4 \left( \frac{3}{\sqrt{2}} \frac{kD^3}{4} \right)^2}{4kD^2 + 2D^2} = \frac{2}{2k + 1} \left( \frac{3}{2} k \right)^{2/3}
\]

When \( H = D \), \( k = 1 \), and \( \phi_s = \left( \frac{3}{2} \right)^{-1/3} = 0.874 \) which is what Geankoplis gives.

For our values of the cylinder dimension:

\[
\phi_s = \frac{4 \left( \frac{3}{\sqrt{2}} \frac{H D^2}{4} \right)^2}{4H \pi D + 2D^2} = 0.8585
\]

\[
D_p = \frac{3}{\sqrt{2}} \frac{H D^2}{4} = 0.0262 \quad (2 \text{ pts})
\]

What this means is that a sphere with a diameter of \( D_p = 0.026 \) has the same volume as a cylinder with diameter \( D = 0.02 \text{ m} \) and height \( H = 0.03 \text{ m} \).

The ratio of surface area to volume is then:

\[
a = a_v (1 - \varepsilon) = 266.67 (1 - 0.3467) = 174.2 \frac{1}{\text{m}} \quad (2 \text{ pts})
\]

**Problem 4.** Geankoplis, problem 3.2-2, page 206

The fluid is air at \( 37.8 \degree C = 311 \text{ K}, D = 0.8 \text{ m}, R = 0.124 \text{ m} \) of water, static pressure is 0.275 m of water gage. The coefficient is 0.97

Calculate the density of air and the static pressure of the stack.
\[
\rho_{\text{air}} = \frac{p_{\text{static}} \cdot MW}{RT}
\]
\[
p_{\text{static}} = p_{\text{atm}} + gh(\rho_{\text{water}} - \rho_{\text{air}})
\]

We have two equations and two unknowns, the density of air and the static pressure. Substitute and solve.

\[
p_{\text{static}} = p_{\text{atm}} + gh\left(\rho_{\text{water}} - \frac{p_{\text{static}} \cdot MW}{RT}\right)
\]

\[
p_{\text{static}} = \frac{p_{\text{atm}} + gh\rho_{\text{water}}}{\left(1 + \frac{ghMW}{RT}\right)} = 104017 \text{ Pa} \quad (2 \text{ pts})
\]

\[
\rho_{\text{air}} = \frac{p_{\text{static}} \cdot MW}{RT} = 1.167 \text{ kg/m}^3 \quad (2 \text{ pts})
\]

\[
v_{\text{max}} = C_p \sqrt{\frac{2(p_2 - p_1)}{\rho}} = 0.97 \sqrt{\frac{2(9.8 \cdot 0.0124 \cdot (1000 - 1.167))}{1.167}}
\]

\[
v_{\text{max}} = 14.0 \text{ m/s} \quad (2 \text{ pts})
\]

\[
N_{Re} = \frac{D\bar{v}\rho}{\mu} = 690,000 \text{ so flow is turbulent. This tells us:}
\]

\[
\bar{v} = 0.86v_{\text{max}} = 12.04 \text{ m/s}
\]

\[
q = \bar{v}A = 6.1 \text{ m}^3/\text{s}
\]

**Problem 5.** Geankoplis, problem 3.2-5, page 207

The fluid is water at 20 C = 293 K, D = 0.0525 m, D2 = 0.020 m, R = 0.214 m of Hg. The coefficient is 0.98
\[ v_2 = \frac{C_v}{\sqrt{1 - \left(\frac{D_2}{D_1}\right)^4}} \sqrt{\frac{2(p_2 - p_1)}{\rho}} \]

\[
= \frac{0.98}{\sqrt{1 - \left(\frac{0.02}{0.0525}\right)^4}} \sqrt{\frac{2(9.8 \cdot 0.214 \cdot (13596 - 1000))}{1000}} \quad (2 \text{ pts})
\]

\[
= 0.99 \cdot 7.27 = 7.20 \, \text{m/s}
\]

\[
q = \bar{v}A = 2.3 \cdot 10^{-3} \, \text{m}^3/\text{s} \quad (2 \text{ pts})
\]

or

\[
\dot{m} = \rho q = 2.3 \, \text{kg/s}
\]

**Problem 6.** Geankoplis, problem 3.2-6, page 207

The fluid is oil at 20 C = 293 K, density = 900 kg/m$^3$, viscosity = 6cp= 0.006 kg/m/s, 
D = 0.1023 m, q=0.0174 m$^3$/s, R = 0.93x10$^5$ Pa. The coefficient is 0.61

\[
v_0 = \frac{q}{A} = \frac{4q}{\pi D_0^2}
\]

\[
v_0 = \frac{C_0}{\sqrt{1 - \left(\frac{D_0}{D_1}\right)^4}} \sqrt{\frac{2(p_1 - p_2)}{\rho}} = \frac{4q}{\pi D_0^2}
\]

\[
\frac{C_0^2}{1 - \left(\frac{D_0}{D_1}\right)^4} \frac{2(p_1 - p_2)}{\rho} = \left(\frac{4q}{\pi D_0^2}\right)^2
\]

\[
\left(\frac{\pi C_0}{4q}\right)^2 \frac{2(p_1 - p_2)}{\rho} = \left(\frac{1}{D_0^2}\right)^2 \left[1 - \left(\frac{D_0}{D_1}\right)^4\right]
\]

\[
\left(\frac{\pi C_0}{4q}\right)^2 \frac{2(p_1 - p_2)}{\rho} = \frac{1}{D_0^4} - \frac{1}{D_1^4}
\]
\[ D_0 = \sqrt[4]{\frac{1}{\left(\frac{\pi C_0}{4q}\right)^2 \left(\frac{2(p_1 - p_2)}{\rho} + \frac{1}{D_1^4}\right)}} = 0.0496\text{m} \]  

(2 pts)

\[ v_0 = \frac{q}{A} = \frac{4q}{\pi D_0^2} = 7.21\text{m/s} \]

\[ v_1 = v_2 = \frac{q}{A} = \frac{4q}{\pi D_0^2} = 2.12\text{m/s} \]

\[ N_{Re,0} = \frac{D_0 v_0 \rho}{\mu} = 53,600 \]

\[ N_{Re,1} = N_{Re,2} = \frac{D_1 v_1 \rho}{\mu} = 23,500 \]

permanent pressure loss:
\[ \frac{D_0}{D_1} = 0.48 \]

So, from Geankoplis, page 132,
\[ \Delta p_{perm} = 0.73(p_2 - p_1) = 67890\text{Pa} \]