Homework Assignment Number Four Solution
Assigned: Wednesday, February 3, 1999
Due: Wednesday, February 10 1999 BEFORE LECTURE STARTS.

Problem 1. Geankoplis, problem 2.8-4, page 110

We have water.

\[ q = 0.050 \text{ m}^3/\text{s} \]

Expanding bend. \( \alpha = 120^\circ \), \( D_1 = 0.0762 \text{m} \), \( D_2 = 0.2112 \text{m} \), \( p_1 = 68,940 \text{Pa(gage)} \)
Neglect energy losses.

\[ \bar{v}_1 = \frac{q}{A_1} = \frac{0.050}{\frac{\pi}{4}D_1^2} = 10.96 \text{ m/s} \]

\[ \bar{v}_2 = \frac{q}{A_2} = \frac{0.050}{\frac{\pi}{4}D_2^2} = 1.43 \text{ m/s} \]

\[ N_{Re1} = \frac{D_1 \bar{v}_1 \rho}{\mu} = \frac{0.0762 \cdot 10.96 \cdot 1000}{0.001} = 8351520 \]

\[ N_{Re2} = \frac{D_2 \bar{v}_2 \rho}{\mu} = \frac{0.2112 \cdot 1.43 \cdot 1000}{0.001} = 302016 \]

turbulent.

Mechanical energy balance:

\[ 0 = \frac{\Delta p}{\rho} + \frac{\Delta v^2}{2\alpha} + g\Delta z + \hat{W}_s + \sum \hat{F} \]

\[ 0 = \frac{p_2 - 68940 - 101325}{1000} + \frac{1.43^2 - 10.96^2}{2} + 0 + 0 + 0 \]

\[ p_2 = 229303 \text{ Pa(abs)} = 127978 \text{ Pa(gage)} \]

From momentum balance on page 74:

\[ R_x = \dot{m}v_2 \cos \alpha_2 - \dot{m}v_1 \cos \alpha_1 + p_2A_2 \cos \alpha_2 - p_1A_1 \cos \alpha_1 \]

\[ R_y = \dot{m}v_2 \sin \alpha_2 - \dot{m}v_1 \sin \alpha_1 + p_2A_2 \sin \alpha_2 - p_1A_1 \sin \alpha_1 + mg \]

\[ \dot{m} = \rho \bar{v}_1 A_1 = \rho \bar{v}_2 A_2 = 50 \text{ kg/s} \]

We must realize that: \( \alpha_1 = 0^\circ \) and \( \alpha_2 = 120^\circ \). Then we know everything to plug into the above equations except \( m_1 \), the total mass inside the pipe, which we cannot calculate without knowing the volume inside the pipe.
Problem 2. Geankoplis, problem 2.8-7, page 110

\[ \bar{v}_1 = 30.5 \frac{m}{s}, \ D_1 = 0.01 \]

Vane is U-shaped so \( \alpha_1 = 0^\circ \) and \( \alpha_2 = 180^\circ \)

Ignore pressure terms for a free jet because pressure is the same everywhere.

\[ R_x = \dot{m}v_2 \cos \alpha_2 - \dot{m}v_1 \cos \alpha_1 \]
\[ R_y = \dot{m}v_2 \sin \alpha_2 - \dot{m}v_1 \sin \alpha_1 + m_t g \]

We ignore the

\[ \dot{m} = \rho \bar{v}_1 A_1 = 1000 \cdot 30.5 \cdot \frac{\pi}{4} \cdot 0.01^2 = 1000 \cdot 30.5 \cdot 7.85 \cdot 10^{-5} = 2.40 \frac{kg}{s} \]

Assume that the cross-sectional area of flow is constant so

\[ \dot{m} = \rho \bar{v}_1 A_1 = \rho \bar{v}_2 A_2 \quad \text{and} \quad \bar{v}_1 = \bar{v}_2, \quad \text{then} \]

\[ R_x = 2.40 \cdot 30.5 \cdot (-1) - 2.40 \cdot 30.5 \cdot (1) = -146.4 \text{N} \]
\[ R_y = 2.40 \cdot 30.5 \cdot (0) - 2.40 \cdot 30.5 \cdot (0) + m_t g = 0 \quad \text{if we neglect gravity.} \]

Problem 3. Geankoplis, problem 2.9-2, page 110

constant density, laminar flow, steady state, horizontal between 2 flat parallel plates. Separated by a distance of \( 2y_0 \).

\[ v_x(y) = \frac{(p_0 - p_L) y_0^2}{2 \mu L} \left[ 1 - \left( \frac{y}{y_0} \right)^2 \right]^2 \quad \text{for} \quad -y_0 < y < y_0 \]

This derivation follows the derivation done in class for the fluid flowing down a plate.

The control volume is a rectangular cube of dimensions

\[ V = LW \Delta y \quad \text{where} \ W \text{ is some arbitrary width in the z-direction,} \ L \text{ is the length in the flow direction,} \ x, \ \text{and} \]

\( \Delta y \) is the incremental width in the direction perpendicular to the parallel plates.

Then the shell momentum balance becomes:
\[ 0 = \Delta y W \tau_{yx} |_{y+\Delta y} - \Delta y W \tau_{yx} |_{y} + \Delta y W \nabla_y (\rho \nabla_y) |_{z=L} - \Delta y W \nabla_y (\rho \nabla_y) |_{z=0} \]

\[ p \Delta y W |_{z=L} - p \Delta y W |_{z=0} \]

The two convective terms in the middle cancel. Divide by \( LW \Delta y \)

\[ \frac{0}{\Delta y} = \frac{\tau_{yx} |_{y+\Delta y} - \tau_{yx} |_{y} + p |_{z=L} - p |_{z=0}}{L} \]

\[ \frac{d\tau_{yx}}{dy} = -\frac{\Delta p}{L} \]

Integrate, using the Boundary Condition given in the problem statement.

\[ \tau_{yx}(y) = -\int_{y=0}^{y} \frac{\Delta p}{L} dy = -\frac{\Delta p}{L} \int_{y=0}^{y} dy \]

\[ \tau_{yx} = -\frac{\Delta p}{L} y \]  \hspace{1cm} (LINEAR PROFILE OF STRESS)

Remember Newton’s Law of Viscosity:

\[ \tau_{yx} = -\mu \frac{dv_x}{dy} \]

\[ -\mu \frac{dv_x}{dy} = -\frac{\Delta p}{\Delta y} y \]

\[ \mu \int_{v_x(y=0)}^{v_x(y)} dv_x = \frac{\Delta p}{L} \int_{y=0}^{y} ydy \]

\[ \mu \int_{v_x(y)}^{v_x(y_0)} dv_x = \frac{\Delta p}{L} \int_{y}^{y_0} ydy \]

\[ \mu (v_x(y_0) - v_x(y)) = \frac{\Delta p}{L} \left( \frac{y_0^2}{2} - \frac{y^2}{2} \right) \]

\[ (0 - v_x(y)) = \frac{\Delta p}{2\mu L} y_o^2 \left( 1 - \frac{y^2}{y_o^2} \right) \]
\[ v_x(y) = -\frac{\Delta p}{2\mu L} y_o^2 \left( 1 - \frac{y^2}{y_o^2} \right) = -\frac{(p_0 - p_L)}{2\mu L} y_o^2 \left( 1 - \frac{y^2}{y_o^2} \right) \]

**PARABOLIC PROFILE**

**Problem 4.** Geankoplis, problem 2.10-1, page 111

Use Hagen-Poiseuille eqn to measure viscosity:

\[ \bar{v} = \frac{\Delta p D^2}{32 \mu L} \] Hagen-Poiseuille Equation,

\[ \mu = \frac{\Delta p D^2}{32 \bar{v} L} \]

\[ \Delta p = \rho g h = 996 \cdot 9.8 \cdot 0.131 = 1289.1 \text{ Pa} \]

\[ \bar{v} = \frac{q}{A} = \frac{q}{\pi \frac{D^2}{4}} = \frac{5.33 \cdot 10^{-7}}{\pi 0.002222^2} = 0.13745 \frac{\text{m}}{\text{s}} \]

\[ \mu = \frac{1289.1 \cdot 0.002222^2}{32 \cdot 0.13745 \cdot 0.1585} = 0.009129 \text{ kg/m/s} \]