Homework Assignment Number Two Solutions
Assigned: Wednesday, January 20, 1999
Due: Wednesday, January 27, 1999 BEGINNING OF CLASS.

Problem 1. Geankoplis, problem 2.3-2, page 105

$$\frac{\partial \Gamma}{\partial t} = \delta \frac{\partial^2 \Gamma}{\partial z^2} + R$$ eqn. (2.3-11), page 42.

No accumulation, no generation.

$$0 = \frac{\partial^2 \Gamma}{\partial z^2} + 0$$

Now, the equation is no longer a PDE but an ODE.

$$\frac{\partial^2 \Gamma}{\partial z^2} = \frac{d^2 \Gamma}{dz^2} = \frac{d\left(\frac{d\Gamma}{dz}\right)}{dz} = 0$$

Integrate out second derivative.

$$\int_{\left(\frac{d\Gamma}{dz}\right)\Big|_{z=z_1}}^{\left(\frac{d\Gamma}{dz}\right)\Big|_{z=z'}} d\left(\frac{d\Gamma}{dz}\right) = \int_{z=z_1}^{z=z'} 0 \cdot dz$$

The integral of zero yields some constant to be determined from the boundary conditions.

$$\left(\frac{d\Gamma}{dz}\right)\Big|_{z=z'} - \left(\frac{d\Gamma}{dz}\right)\Big|_{z=z_1} = 0$$

Integrate out first derivative.

$$\int_{\Gamma(z'=z)}^{\Gamma(z'=z_2)} d\Gamma = \int_{z'=z}^{z'=z_2} \left[\left(\frac{d\Gamma}{dz}\right)\Big|_{z=z_1}\right] dz'$$

$$\Gamma(z' = z_2) - \Gamma(z' = z) = \left(\left(\frac{d\Gamma}{dz}\right)\Big|_{z=z_1}\right)(z_2 - z)$$

$$\Gamma(z) = \Gamma_2 - \left(\left(\frac{d\Gamma}{dz} \right) \Bigg|_{z=z_1} \right) (z_2 - z)$$

at z_1,

$$\Gamma(z_1) = \Gamma_1 = \Gamma_2 - \left(\left(\frac{d\Gamma}{dz} \right) \Bigg|_{z=z_1} \right) (z_2 - z_1)$$

so, by rearranging:

$$\left(\frac{d\Gamma}{dz} \right) \Bigg|_{z=z_1} =$$

and substituting this result back into the equation for gamma:

$$\Gamma(z) = \Gamma_2 - \left(\frac{\Gamma_2 - \Gamma_1}{z_2 - z_1} \right) (z_2 - z)$$

This equation describes a straight line between the boundary points.

Problem 2. Geankoplis, problem 2.4-2, page 105

Using figure 2.4-1. v=0.4m/s. Fluid is water. T= 24 C.

(a) Using equation (2.4-5), we have $\tau_{yz} = -\mu \dfrac{v_z(y_2) - v_z(y_1)}{y_2 - y_1}$

$$\tau_{yz} = -\left(.9142 \cdot 10^{-3}\right) \frac{0.4 m/s}{\Delta y} = -0.3 N/m^2$$

$$\Delta y = \left(.9142 \cdot 10^{-3}\right) \frac{0.4 m/s}{.3 N/m^2} = 1.2 \cdot 10^{-3} m$$

shear rate = $\dfrac{v_z(y_2) - v_z(y_1)}{y_2 - y_1}$ = 333.33 sec^{-1}

(b) oil

$$\tau_{yz} = -(0.02) \frac{0.4}{.0012} = -6.6667 \frac{N}{m^2}$$

shear rate = $\dfrac{v_z(y_2) - v_z(y_1)}{y_2 - y_1}$ = 333.33 sec^{-1}

Problem 3. Geankoplis, problem 2.5-1, page 105

(a) $$A = \left(\frac{\pi}{4}D^2\right) = \frac{\pi}{4}\left(63.5\text{mm} \cdot \frac{1\text{m}}{1000\text{mm}}\right)^2 = 0.00317\text{m}^2$$

$$\dot{m} = \rho A v$$

$$v = \frac{\dot{m}}{\rho A} = \frac{0.605\frac{\text{kg}}{\text{s}}}{1030\frac{\text{kg}}{\text{m}^3} \cdot 0.00317\text{m}^2} = 0.185\frac{\text{m}}{\text{s}}$$

$$N_{Re} = \frac{Dv\rho}{\mu} = \frac{\left(63.5\text{mm} \cdot \frac{1\text{m}}{1000\text{mm}}\right)\left(0.185\frac{\text{m}}{\text{s}}\right)\left(1030\frac{\text{kg}}{\text{m}^3}\right)}{2.12\text{cp} \cdot \frac{10^{-3}\frac{\text{kg}}{\text{m} \cdot \text{s}}}{\text{cp}}} = 5710$$

TURBULENT

(b) $$N_{Re} = \frac{Dv\rho}{\mu} = \frac{(0.0635\text{m})\left(v\frac{\text{m}}{\text{s}}\right)\left(1030\frac{\text{kg}}{\text{m}^3}\right)}{0.00212\frac{\text{kg}}{\text{m} \cdot \text{s}}} = 2100$$

$$v = 0.0681\frac{\text{m}}{\text{s}}$$

$$q = Av = 0.00317\text{m}^2 \cdot \left(0.0681\frac{\text{m}}{\text{s}}\right) = 2.16 \cdot 10^{-4}\frac{\text{m}^3}{\text{s}}$$

Problem 4. Geankoplis, problem 2.6-1, page 106

$$v_z = \frac{\rho g \delta^2}{2\mu}\left[1 - \left(\frac{x}{\delta}\right)^2\right]$$

(a) $v_{z,max} = \dfrac{\rho g \delta^2}{2\mu}$ at x = 0, by inspection, or:

Take the derivative with respect to x and set it equal to zero. Solving this for the velocity, will give the extrema of the velocity.

(b) $\bar{v} = \frac{\iint_A v dA}{\iint_A dA} = \frac{1}{A}\iint_A v dA$

which for free flow down a vertical plate of width, W, becomes

$$\bar{v} = \frac{1}{W\delta}\int_0^{\delta}\int_0^{W} v_{max}\left[1-\left(\frac{x}{\delta}\right)^2\right]dydx$$

There is no y-dependence on the velocity (across the width of the plate) so that integrates out as a W and cancels with the W in the denominator.

$$\bar{v} = \frac{W}{W\delta}\int_0^{\delta} v_{max}\left[1-\left(\frac{x}{\delta}\right)^2\right]dx$$

$$\bar{v} = \frac{v_{max}}{\delta}\int_0^{\delta}\left[1-\left(\frac{x}{\delta}\right)^2\right]dx = \frac{v_{max}}{\delta}\left[x-\frac{x^3}{3\delta^2}\right]_0^{\delta} = \frac{v_{max}}{\delta}\left[\delta-\frac{\delta^3}{3\delta^2}\right]$$

$$\bar{v} = \frac{v_{max}}{\delta}\left[\delta-\frac{\delta}{3}\right] = \frac{2v_{max}}{3\delta}\delta = \frac{2v_{max}}{3}$$

Problem 5. Geankoplis, problem 2.6-2, page 106

$$\dot{m}_{in} = \rho A v = \left(902\frac{kg}{m^3}\right)\left(0.00433 m^2\right)\left(1.282\frac{m}{s}\right) = 5.01\frac{kg}{s}$$

$\dot{m}_{out} = \dot{m}_{in}$ by conservation of mass and the steady state assumption.

$$v_{out} = \frac{\dot{m}}{\rho A} = \frac{5.01\frac{kg}{s}}{875\frac{kg}{m^3}\cdot 0.00526 m^2} = 1.09\frac{m}{s}$$

$$massflux_{in} = \frac{\dot{m}}{A} = \frac{5.01\frac{kg}{s}}{0.00433 m^2} = 1156\frac{kg}{s\cdot m^2}$$