Homework Assignment Number One
Assigned: Wednesday, January 13, 1999
Due: Wednesday, January 20, 1999 BEGINNING OF CLASS.

Problem 1. Unit Conversions between concentrations and densities
Water has an approximate (mass) density of 1.0 grams/ml and a molecular weight of 18.0 grams/mole.
(a) What is the molarity (molar density or molar concentration, or simply concentration) of water in mol/liter?
(b) What is the mass density of water in kg/m^3?
(c) What is the molar volume of water in liter/mol?

Air has an approximate molecular weight of 28.84 grams/mol and roughly obeys the ideal gas law at ambient conditions, P = 101325 Pa and T = 298 K.
(d) What is the molar density of air at these conditions in mol/liter?
(e) What is the mass density of air at these conditions in kg/m^3?

Solution:
(a)

$$\tilde{n} = C = \frac{\rho}{MW} = \frac{1.0\,\frac{g}{ml}}{18.0\,\frac{g}{mol}} = 0.05556\,\frac{mol}{ml}\cdot\frac{1000ml}{liter} = 55.556\,\frac{mol}{liter}$$

(b)

$$\rho = 1.0\,\frac{g}{ml}\cdot\frac{kg}{1000g}\cdot\frac{10^6 ml}{m^3} = 1000.0\,\frac{kg}{m^3}$$

(c)

$$\tilde{V} = \frac{1}{C} = \frac{1}{55.556\,\frac{mol}{liter}} = 0.018\,\frac{liter}{mol}$$

(d)

$$\tilde{n} = \frac{n}{V} = \frac{P}{RT} = \frac{101325Pa}{8.314\,\frac{J}{mol\cdot K}\cdot 298K} = 40.897\,\frac{mol}{m^3}\cdot\frac{m^3}{1000.0l} = 0.0409\,\frac{mol}{l}$$

(e)

$$\rho = \tilde{n}\cdot MW = 40.897\,\frac{mol}{m^3}\cdot\frac{18g}{mol}\cdot\frac{kg}{1000.0l} = 0.736\,\frac{kg}{m^3}$$

Problem 2. Dimensionally Consistent Equations
For the following balance equations, (i) write down the units of each term in the equation, (ii) state whether the equation is dimensionally consistent, (iii) correct the equation to be dimensionally consistent.
V is the volume, C_A is the molar concentration of component A, t is the time, F_i is the molar flowrate of the ith stream, $\hat{F}_i$ is the mass flowrate of the ith stream, $x_{i,A}$ is the mole fraction of component A in the ith stream,

$\hat{x}_{i,A}$ is the mass fraction of component A in the i^th stream, k is a first order reaction rate constant with units of inverse time, ρ_A is the mass density of component A, MW_A is the molecular weight of component A, C_{p_A} is the molar heat capacity of component A, $\hat{C}_{p_A}$ is the mass heat capacity of component A, T_i is the temperature of the i^th stream, n is the number of components in the system, ΔH_r is the molar heat of reaction.

Solution:

(a) mole balance: $V\frac{dC_A}{dt} = F_{in}x_{in,A} - F_{out}x_{out,A} - kVC_A$

mol/time = mol/time - mol/time - mol/time

The equation is dimensionally consistent as written.

(b) mass balance: $V \cdot MW_A \frac{dC_A}{dt} = \hat{F}_{in}\hat{x}_{in,A} - MW_A\hat{F}_{out}x_{out,A} - kV\rho_A$

mass/time = mass/time - mass^2/mol/time - mass/time

The equation is not dimensionally consistent as written.

The equation would be correct if written as:

$$V \cdot MW_A \frac{dC_A}{dt} = \hat{F}_{in}\hat{x}_{in,A} - \hat{F}_{out}\hat{x}_{out,A} - kV\rho_A$$

(c) energy balance:

$$V\sum_{i=j}^{n}\frac{d\left(C_{p_j}C_j\left(T - T_{ref}\right)\right)}{dt} = F_{in}\sum_{i=j}^{n}C_{p_j}x_{in,j}\left(T_{in} - T_{ref}\right)$$

$$- F_{out}\sum_{i=j}^{n}\hat{C}_{p_j}\rho_j MW_j\left(T_{out} - T_{ref}\right) - \Delta H_r kC_A$$

energy/time = energy/time - energy*mass/volume/time - energy/time/volume

The equation is not dimensionally consistent as written.

The equation would be correct if written as:

$$V\sum_{i=j}^{n}\frac{d\left(C_{p_j}C_j\left(T - T_{ref}\right)\right)}{dt} = F_{in}\sum_{i=j}^{n}C_{p_j}x_{in,j}\left(T_{in} - T_{ref}\right)$$

$$- F_{out}\sum_{i=j}^{n}C_{p_j}x_{out,j}\left(T_{out} - T_{ref}\right) - \Delta H_r kVC_A$$

Problem 3. Pressure Conversions

Convert a pressure of 100 psia to absolute pressure in

(a) Pa, (b) bar, (c) atmospheres, (d) feet of water, (e) feet of mercury (use specific gravity in text).

(a) $100.0\text{psia} \cdot \frac{1.0\text{atm}}{14.7\text{psia}} \cdot \frac{101325\text{Pa}}{1.0\text{atm}} = 689300\text{Pa}$

(b) $$689300\text{Pa} * \frac{1\text{bar}}{10^5\text{Pa}} = 6.893\text{bar}$$

(c) $$100.0\text{psia} \cdot \frac{1.0\text{atm}}{14.7\text{psia}} = 6.80\text{atm}$$

(d) $$h = \frac{pg_c}{\rho g} = \frac{689300\text{Pa}}{1000\frac{\text{kg}}{\text{m}^3} \cdot 9.8\frac{\text{m}}{\text{s}^2}} = 70.34\text{m} \cdot \frac{3.2808\text{ft}}{\text{m}} = 230.8\text{ft}$$

(e) $$h = \frac{pg_c}{\rho g} = \frac{689300\text{Pa}}{\left(1000\frac{\text{kg}}{\text{m}^3} \cdot 13.5955\right) \cdot 9.8\frac{\text{m}}{\text{s}^2}} = 5.17\text{m} \cdot \frac{3.2808\text{ft}}{\text{m}} = 16.97\text{ft}$$

Problem 4. U-Tube Manometers
Geankoplis 2.2-4, pg. 104.

$$p_b = 754\text{mmHg},\ \rho_a = 1000\frac{\text{kg}}{\text{m}^3},\ \rho_b = 1.3\frac{\text{kg}}{\text{m}^3}, Z \approx 0.0\text{m},\ R = 0.415\text{m}$$

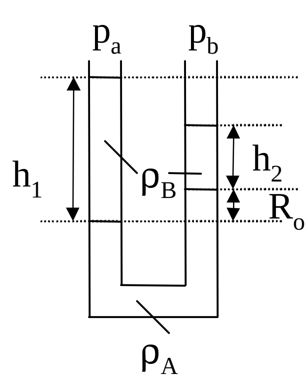
$$p_a = p_b + R(\rho_A - \rho_B)\frac{g}{g_c} = \frac{754}{760}101325 + 0.415 * (1000 - 1.3) * 9.8$$

$$p_a = 104600\text{Pa} = 104.6\text{kPa}$$

$$p_a = 104600\text{Pa}\frac{1.0\text{atm}}{101325\text{Pa}} \cdot \frac{14.7\text{psia}}{1.0\text{atm}} = 15.17\text{psia}$$

Problem 5. two-fluid U-tube Manometers with reservoirs
Geankoplis gives a diagram of a two-fluid U-tube manometer with wide reservoirs in figure 2.2-4(b) on page 36. He gives the working equation for it in equation 2.2-15 on page 37. However, he does not provide a derivation for the equation. Moreover, it is not a completely straightforward derivation. I would like you to derive equation 2.2-15. To do this we will take several steps.

(a) Consider:



R_o is the initial difference between fluid levels at zero pressure difference. If the pressure at point a is the same as the pressure at point b but the amount of B-fluid in the two legs of the manometer is different (so $h_1 \neq h_2$), find R_o in terms of h_1, h_2, ρ_A, ρ_B.

Solution:

The pressure at the bottom dotted line on each leg must be equal due to the principle of fluid statics.

$$p_1 = p_2$$
$$\rho_B g h_1 = \rho_B g h_2 + \rho_A g R_o$$
$$R_o = \frac{\rho_B}{\rho_A}(h_1 - h_2)$$

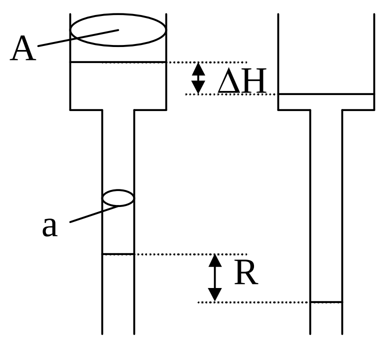
(b) In the above figure, if the actual reading is R. What is the pressure difference between points a and b in terms of R, R_o, ρ_A, ρ_B?

Solution:

$$p_a - p_b = (R - R_o)(\rho_A - \rho_B)\frac{g}{g_c}$$

We must change our standard formula to include R_o since the manometer does not read zero at zero pressure difference.

(c) Consider some arbitrary change in pressure that cause a shift in a fluid from the figure on the left to the figure on the right.



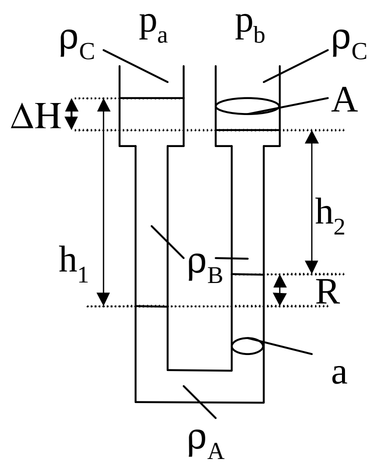
where a and A are cross-sectional areas. Express ΔH in terms of R, a, A. (The underlying principle here is conservation of volume. The volumes in both tubes are equal.)

Solution:

$$Volume_1 = Volume_2$$
$$V_{1,TOP} + V_{1,BOT} = V_{2,TOP} + V_{2,BOT}$$
$$V_{1,TOP} - V_{2,TOP} = V_{2,BOT} - V_{1,BOT}$$
$$A\Delta H = aR$$

$$\Delta H = \frac{a}{A} R$$

(d) Now, considering the correction factor from parts (a-b) for a non-zero default reading and considering the factor for a change in tube cross-sectional area, derive equation 2.2-15 for the two-fluid U-Tube manometer with reservoirs.



Solution:

$$p_a - p_b = (R - R_o)(\rho_A - \rho_B)\frac{g}{g_c} + \frac{a}{A}(R - R_o)(\rho_B - \rho_C)\frac{g}{g_c}$$

On the right-hand-side of the equation above, the first term accounts for the difference in height of the A-B fluid interface in the two legs of the manometer and for the correction term due to a non-zero equilibrium reading. The second term accounts for the difference in height of the B-C fluid interface in the two reservoirs of the manometer and for a correction term analogous to the one obtained in the legs. Rearranging yields:

$$p_a - p_b = (R - R_o)\left(\rho_A - \rho_B + \frac{a}{A}\rho_B - \frac{a}{A}\rho_C\right)\frac{g}{g_c}$$

(e) Geankoplis says on page 38 that, for the two-fluid U-Tube manometer with reservoirs, "R_o is often adjusted to zero". How is this done?

Solution:

The amount of B-fluid in each of the reservoirs must be the same. (look at the result from part (a). What will give a zero R_o? When $h_1 = h_2$, which is equivalent to saying that the amount of B-fluid in each of the reservoirs is the same.