Problem 1. Fluid Statics

When the difference in densities of two immiscible fluids is not great, separation by gravity in a decanter is not feasible. We can increase the drive toward separation by increasing the acceleration. To do this, we rely on the centrifugal force of a centrifuge rather than the force of gravity. Consider the centrifuge shown below, employed to separate 2 immiscible fluids A and B where \( \rho_A > \rho_B \). If the radial acceleration of the centrifuge is \( a = \omega^2 r \) where \( \omega \) is the angular velocity in radians/sec and \( r \) is the position from the center of the centrifuge, then the pressure drop due to the centrifuge is given by:

\[
\Delta p = p_2 - p_1 = \rho_B \omega^2 \left( \frac{r_2^2 - r_1^2}{2} \right)
\]

Using this fact, derive an equation for the location of the A-B interface, \( r_i \) in terms of \( \omega \), \( \rho_A \), \( \rho_B \), and the overflow positions, \( r_A \), and \( r_B \). Neglect gravity. Carefully indicate on the diagram the location of the two points where you are calculating the pressure to be used in the principle of fluid statics.

Solution:

The pressure at the interface anywhere in the centrifuge is

\[
p_i = \frac{\rho_B \omega^2 (r_i^2 - r_B^2)}{2} + p_o
\]

The pressure in the A-fluid at the same radial position but located inside the A-overflow duct is
Using the principle of fluid statics, equate the interface pressures.

\[ p_i = \frac{\rho_A \omega^2 (r_i^2 - r_A^2)}{2} + p_o = \frac{\rho_B \omega^2 (r_i^2 - r_B^2)}{2} + p_o \]

Solve for \( r_i \).

\[ \rho_A (r_i^2 - r_A^2) = \rho_B (r_i^2 - r_B^2) \]

\[ (\rho_A - \rho_B) r_i^2 = \rho_A r_A^2 - \rho_B r_B^2 \]

\[ r_i = \sqrt{\frac{\rho_A r_A^2 - \rho_B r_B^2}{\rho_A - \rho_B}} \]

### Problem 2. Mass balance and mechanical energy balance

Consider the following flow system:

\[
\begin{align*}
&1, D_1, z_1, p_1 \\
&\downarrow\quad w_p \\
&2, D_2, z_2, p_2
\end{align*}
\]

where the fluid is water. Use a density, \( \rho = 1000.0 \text{ kg/m}^3 \) and a viscosity, \( \mu = 1.0 \text{ cp} \). Assume steady state. The diameters of the lines are: \( D_1 = 7.6 \text{ cm} \) and \( D_2 = 5.1 \text{ cm} \). The elevations of the lines are: \( z_1 = 0.0 \text{ m} \) and \( z_2 = 5.0 \text{ m} \). The pressure at the feed to the pump is \( p_1 = 1.0 \text{ atm} \) and the pressure at the outlet is \( p_2 = 1.0 \text{ atm} \). The volumetric flow rate feeding into the pump is \( q_1 = 0.002 \text{ m}^3/\text{s} \). The power supplied to the pump is \( \dot{W}_p = 3.0 \text{ hp} \). The efficiency of the pump is 80%.

(a) Calculate the inlet and exit velocities in m/s
(b) Calculate total friction loss in J/kg.

**Solution:**
acc = in – out +/− gen / con
\[ 0 = \dot{m}_1 - \dot{m}_2 + 0 \]
\[ \dot{m}_2 = \rho \nabla_2 A_2 = \dot{m}_1 = \rho \nabla_1 A_1 = \rho q_1 = 1000.0 \cdot 0.002 = 2 \text{ kg/sec} \]
\[ \bar{v}_1 = \frac{\dot{m}_1}{\rho A_1} = \frac{\dot{m}_1}{\rho \frac{\pi D_1^2}{4}} = \frac{2}{1000 \frac{\pi}{4} 0.076^2} = 0.4409 \text{ m/s} \]
\[ \bar{v}_2 = \frac{\dot{m}_2}{\rho A_2} = \frac{\dot{m}_2}{\rho \frac{\pi D_2^2}{4}} = \frac{2}{1000 \frac{\pi}{4} 0.051^2} = 0.9790 \text{ m/s} \]

(b) The equations we need are:
\[ 0 = \frac{\Delta p}{\rho} + \frac{\Delta v^2}{2 \alpha} + g\Delta z + \dot{W}_s + \sum \dot{F} \]

and
\[ W_S = -\eta W_P \]

To determine which value of \( \alpha \) to use, calculate the Reynold’s number.
\[ N_{Re,1} = \frac{D_1 \bar{v}_1 \rho}{\mu} = \frac{0.076 \cdot 0.4409 \cdot 1000}{0.001} = 33,500 \]
\[ N_{Re,2} = \frac{D_1 \bar{v}_1 \rho}{\mu} = \frac{0.051 \cdot 0.9790 \cdot 1000}{0.001} = 49,900 \]

Flow is turbulent. Use \( \alpha = 1 \).

Now calculate \( \dot{W}_S \)
\[ \dot{W}_S = -\eta \dot{W}_P = -0.8 \cdot 3.0 \text{ hp} \frac{745.7 \text{ W}}{\text{ hp}} = -1789.7 \frac{\text{ J}}{\text{ s}} \]
\[ \dot{W}_S = \frac{\dot{W}_S}{\dot{m}} = \frac{-1789.7 \frac{\text{ J}}{\text{ s}}}{2 \frac{\text{ kg}}{\text{ s}}} = -894.8 \frac{\text{ J}}{\text{ kg}} \]
\[ 0 = \frac{0 + 0.9790^2 - 0.4409^2}{2} + 9.8(5.0 - 0.0) - 894.8 + \sum \dot{F} \]
\[ \sum \dot{F} = 845.5 \frac{\text{ J/kg}}{\text{ kg}} \]
Problem 3. Mass balance, mechanical energy balance, and fluid statics
Consider the system shown below, where a pressurized line is split into two lines that empty into the same, open storage tank. The level in the tank is maintained by an overflow.

The fluid is water. Use a density, $\rho = 1000.0 \frac{kg}{m^3}$. Assume steady state. Neglect friction. Neglect kinetic energy terms. The diameters of the three lines are: $D_1 = 7.6\text{cm}$, $D_2 = 5.1\text{cm}$, $D_3 = 6.4\text{cm}$. The elevations of the three lines are: $z_1 = 0.0\text{m}$, $z_2 = 31.0\text{m}$, $z_3 = -6.0\text{m}$. The height of the overflow in the tank is $h = 10.0\text{m}$. The pressure at the pump is $p_1 = 2.0\text{atm}$ and the velocity at the pump is $\bar{V}_1 = 0.10\text{m/s}$. What fraction of the total flow in line #1 enters the tank through line #3? You must write down the complete mass and energy balances.

Solution:

Mass balance:

$$\text{acc } = \text{in } - \text{out } + \text{gen } / \text{con}$$
$$0 = \dot{m}_1 - \dot{m}_2 - \dot{m}_3 + 0$$
$$0 = \rho \bar{V}_1 A_1 - \dot{m}_2 - \dot{m}_3$$
$$\dot{m}_2 = \rho \bar{V}_1 \frac{\pi}{4} D_1^2 - \dot{m}_3$$

Mechanical energy balance:

$$\text{acc } = \text{in } - \text{out } + \text{gen } / \text{con}$$

$$\frac{\partial}{\partial t} \left[ \rho \bar{V} \left( U + \frac{v^2}{2} + zg \right) \right] = \left[ \rho \bar{V} A \left( H + \frac{v^2}{2\alpha} + zg \right) \right]_{\text{in}} - \left[ \rho \bar{V} A \left( H + \frac{v^2}{2\alpha} + zg \right) \right]_{\text{out}} + q - \dot{W}_s$$

Ignore kinetic energy term:

$$0 = [\dot{m}_1 (H + zg)]_1 - [\dot{m}_2 (H + zg)]_2 - [\dot{m}_3 (H + zg)]_3 + q - \dot{W}_s$$
Following the derivation from lecture, for the definition of the enthalpy:

\[
0 = -m_2 \left[ \frac{\Delta p}{\rho} + g \Delta z \right]_{21} - m_3 \left[ \frac{\Delta p}{\rho} + g \Delta z \right]_{31} - \dot{W}_s - \sum F
\]

Neglecting friction and in the absence of shaft work, we have:

\[
\dot{m}_2 \left[ \frac{\Delta p}{\rho} + g \Delta z \right]_{21} = -\dot{m}_3 \left[ \frac{\Delta p}{\rho} + g \Delta z \right]_{31}
\]

We know the three elevations and we know the three pressures, because

\[p_2 = 1.0 \text{ atm}\]

\[p_3 = h\rho g + p_2\]

So we have 2 equations, the mass balance and the energy balance, and two unknowns, the flowrates of the two exit streams. Solve. Rearrange energy balance, equate to mass balance.

\[
\dot{m}_2 = -\dot{m}_3 \frac{\Delta p}{\rho + g \Delta z} \frac{31}{21} = \rho \bar{v}_1 \frac{\pi}{4} D_1^2 - \dot{m}_3
\]

\[
\dot{m}_3 = \left( 1 - \frac{\Delta p}{\rho + g \Delta z} 31 \right) \frac{\rho \bar{v}_1 \frac{\pi}{4} D_1^2}{21}
\]

\[
\dot{m}_3 = \frac{0.4536}{1 - \frac{-62.125}{202.475}} = 0.3471 \text{ kg/sec}
\]

\[
\dot{m}_1 = \rho \bar{v}_1 \frac{\pi}{4} D_1^2 = 1000 \cdot 0.1 \cdot \frac{\pi}{4} \cdot 0.076^2 = 0.4536 \text{ kg/sec}
\]
\[ \frac{m_3}{m_1} = \frac{0.3471}{0.4536} = 0.765 \]