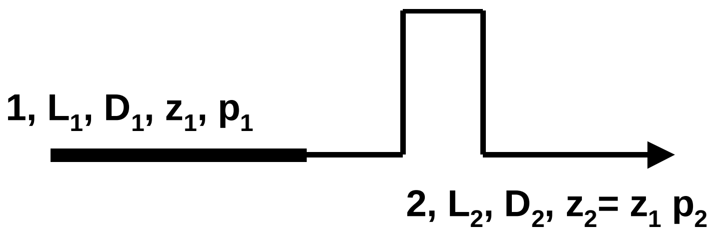
Examination Number Two
Administered: Friday, March 5, 1999

Problem 1. Friction and mechanical energy balance

Consider the following flow system:



where the fluid is water. Use a density, $\rho = 1000.0 \frac{kg}{m^3}$ and a viscosity, $\mu = 1.0cp$. Assume steady state. The diameters of the lines are: $D_1 = 0.076m$ and $D_2 = 0.051m$. Thelengths of the lines are: $L_1 = 10.0m$ and $L_2 = 30.0m$.The elevations of the lines are: $z_1 = 0.0m$ and $z_2 = z_1$. The volumetric flow rate feeding into the pump is $q_1 = 0.002 \frac{m^3}{s}$.

(a) List and calculate all frictional terms in the mechanical energy balance. Calculate the total frictional loss. State all answers in [J/kg].

For your convenience, use the pre-calculated values:

$$\bar{v}_1 = \frac{q}{A_1} = 0.44 \frac{m}{s} \qquad \bar{v}_2 = \frac{q}{A_2} = 0.98 \frac{m}{s}$$

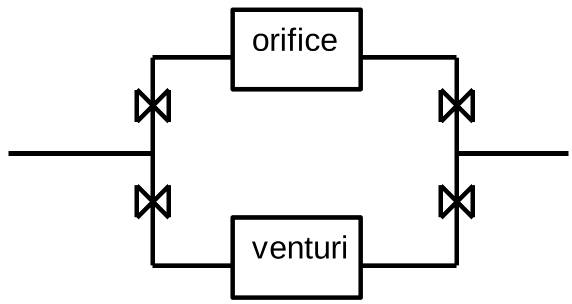
$$N_{Re,1} = \frac{D_1 \bar{v}_1 \rho}{\mu} = 33400 \qquad N_{Re,2} = \frac{D_2 \bar{v}_2 \rho}{\mu} = 50000$$

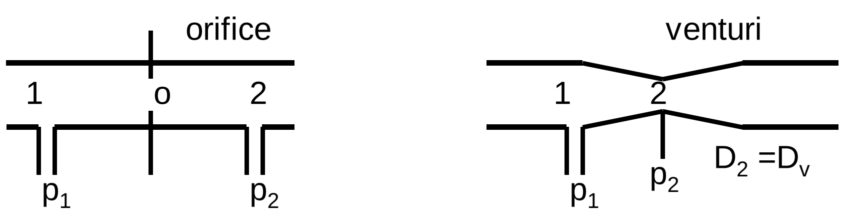
$$\frac{\varepsilon}{D_1} = 0.0006 \qquad \frac{\varepsilon}{D_2} = 0.0009$$

(b) Calculate the pressure drop in the system. State your answer in [Pa].

Problem 2. Flow Measurement

Consider the following flow system, designed to measure flow with either an orifice or a venturi meter, depending on whether the valves at the top or bottom are open. The pressure loss across either meter can be divided into two terms: a permanent pressure loss and a recoverable pressure loss, where some of the mechanical energy lost to the formation of vortices is reclaimed when the vortices dissipate downstream of the meters. The fraction of measured pressure loss that is permanent is approximately 10% for the venturi meter and 73% for the orifice. Find the orifice diameter that will give the same permanent pressure loss as the venturi throat diameter. Express you answer as $D_o = f(D_v, C_o, C_v, D_1)$. Is D_o greater than, less than, or equal to D_v?





(Hint: remember $\dot{m} = \rho \bar{v} A$ so $\bar{v} = \frac{\dot{m}}{\rho A} = \frac{4\dot{m}}{\rho \pi D^2}$)

Problem 3. Navier-Stokes Equation

An incompressible, Newtonian fluid flows down the side of a vertically upright steel plate. Let x designate the vertical dimension, y is a lateral dimension perpendicular to the face of the plate, and z is a lateral dimension parallel to the face of the plate. Consider one-dimensional flow down the plate where the fluid film has a thickness in the y-direction of y_o.

(a) The Navier-Stokes equation can be written for the x, y, and z component of momentum. Which one component is of interest in this one-dimensional-flow problem? Why?

(b) For the component you selected in part (a) write out the full Navier-Stokes equation (don't use substantial derivatives).

(c) For the equation you wrote in part (b), cross-out all negligible terms. Explain your reasoning in deleting terms. Is the resulting equation an ordinary differential equation or a partial differential equation?

(d) Our boundary condition at the free interface is: $\frac{dv_x}{dy}(y = y_o) = 0$.

Derive the shear rate profile, ($\frac{dv_x}{dy}$ as a function of y.)

(e) Our boundary condition at the wall is: $v_x(y=0)=0$

Derive the velocity profile, (v_x as a function of y.)

Problem 4.

Explain with words or sketches laminar and turbulent flow.