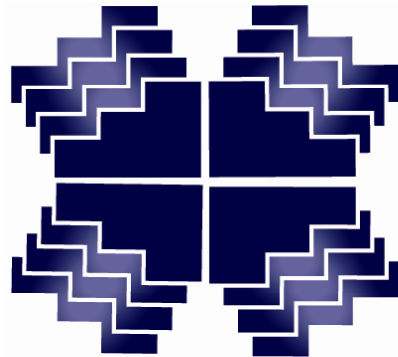


# Multiscale Materials Modeling

## Lecture 05

### Proton Transport in Aqueous Systems



# STAIR

Sustainable Technology through  
Advanced Interdisciplinary Research

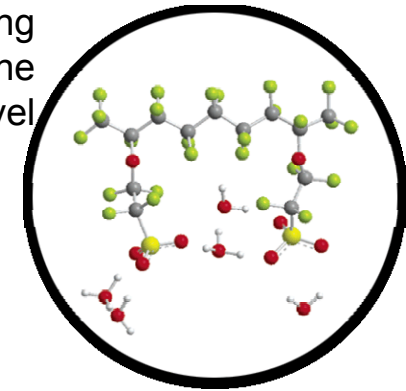
## Proton Transport in Aqueous Systems

- I. Introduction
- II. Levels of Modeling
  - II.A. Quantum Mechanics
  - II.B. Reactive Molecular Dynamics
  - II.C. Confined Random Walk Theory
  - II.D. Percolation Theory
- III. Conclusions

# Moving toward fuel cell-powered vehicles



understanding starts at the quantum level



H<sub>2</sub>-powered autos become a reality

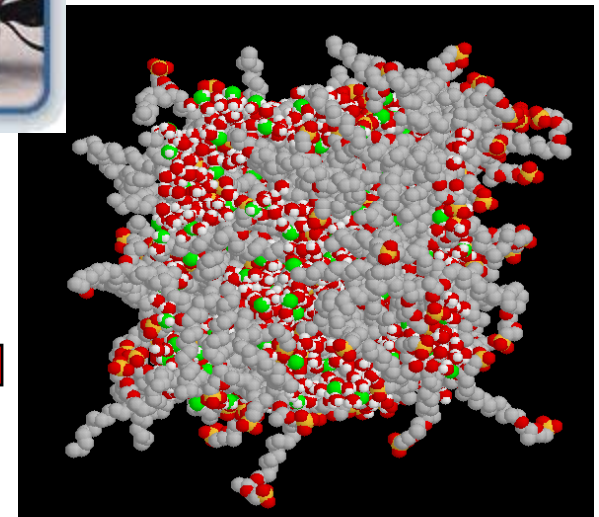


leads to high-fidelity coarse-grained models

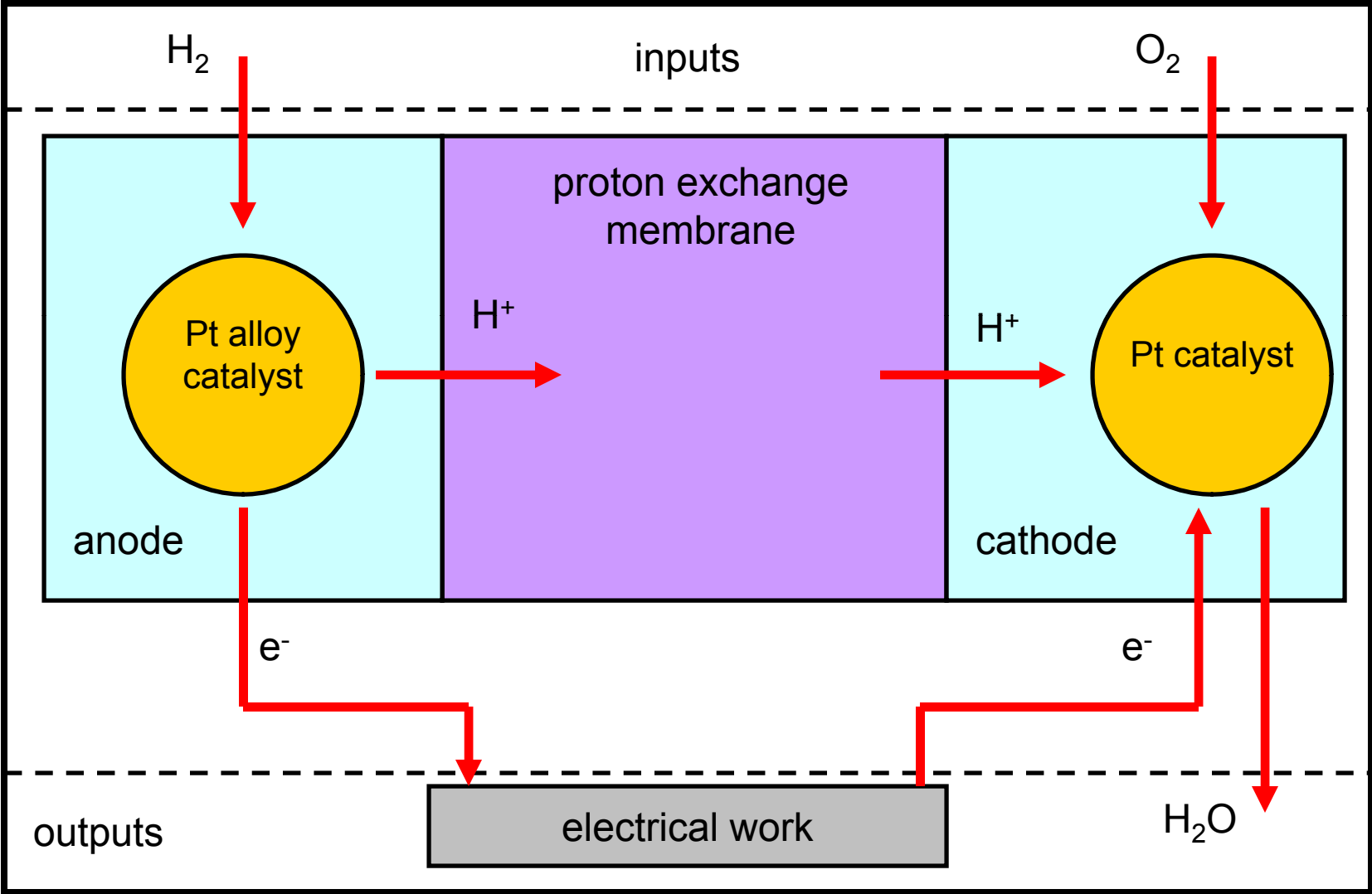


impacts fuel cell performance

improved nanoscale design of membrane/electrode assembly



# how fuel cells work: conceptual level

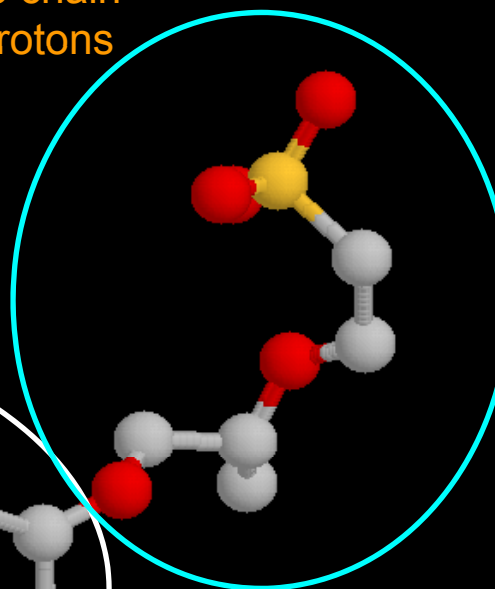
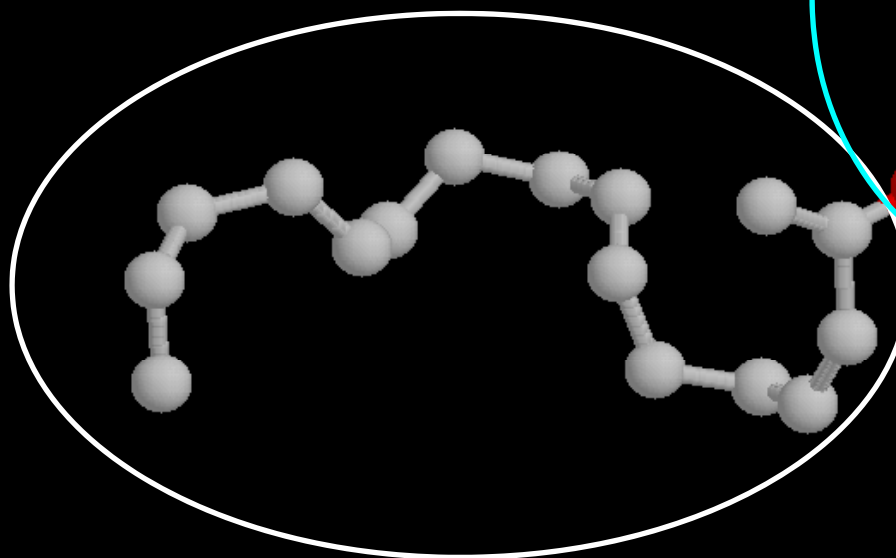


# proton exchange membranes are polymer electrolytes

industry standard:  
Nafion (DuPont)  
perfluorosulfonic acid

sulfonic acid at  
end of side chain  
provides protons

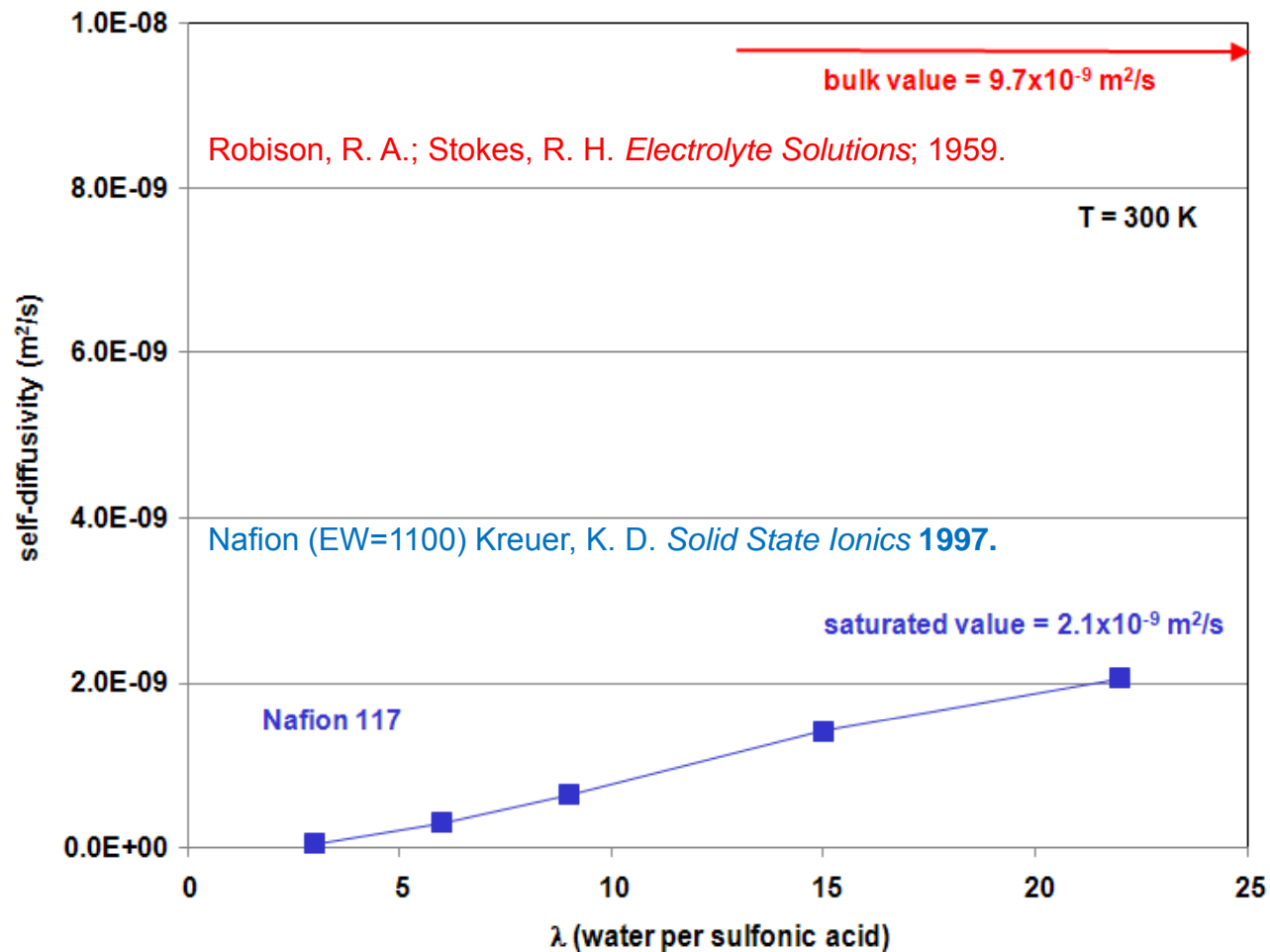
monomer backbone contains  $\text{CF}_2$ .



side chain

$\text{CF}_2$  = gray, O = red, S = orange, cation not shown.

# Proton Transport in Bulk Water and PEM Experimental Measurements



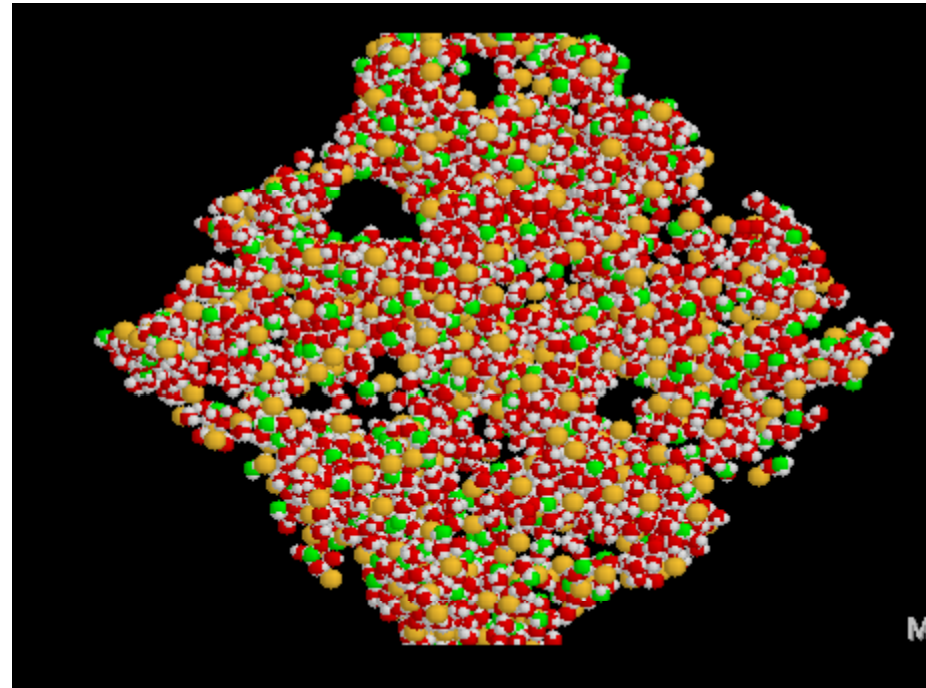
Even at saturation, the self-diffusivity of charge in Nafion is 22% of that in bulk water.

# morphology of bulk hydrated membrane

Nafion

EW = 1144  
 $\lambda = 6 \text{ H}_2\text{O}/\text{HSO}_3$   
T = 300 K

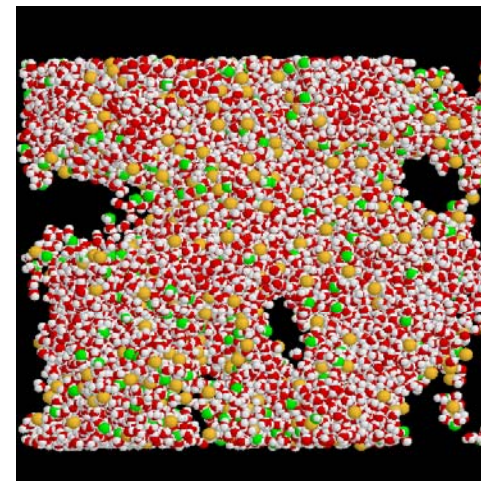
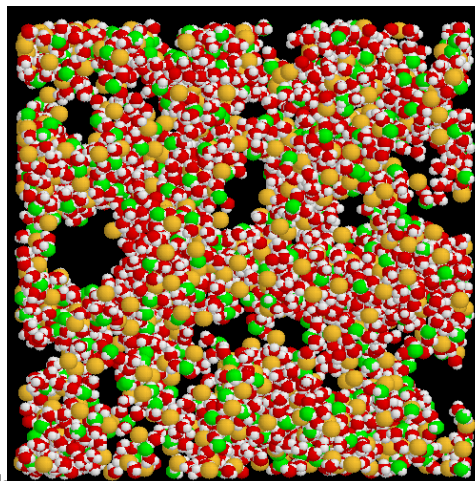
Snapshots of the aqueous nanophase



legend:  
O of  $\text{H}_2\text{O}$  = red  
H = white  
O of  $\text{H}_3\text{O}^+$  = green  
S = orange  
remainder of polymer electrolyte not shown

PEM morphology is a function of water content.

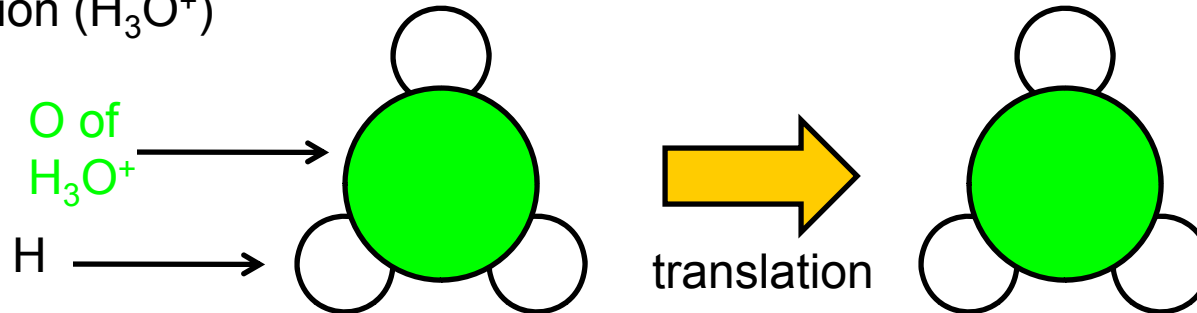
low water ( $\lambda = 6$ )  
small aqueous channels



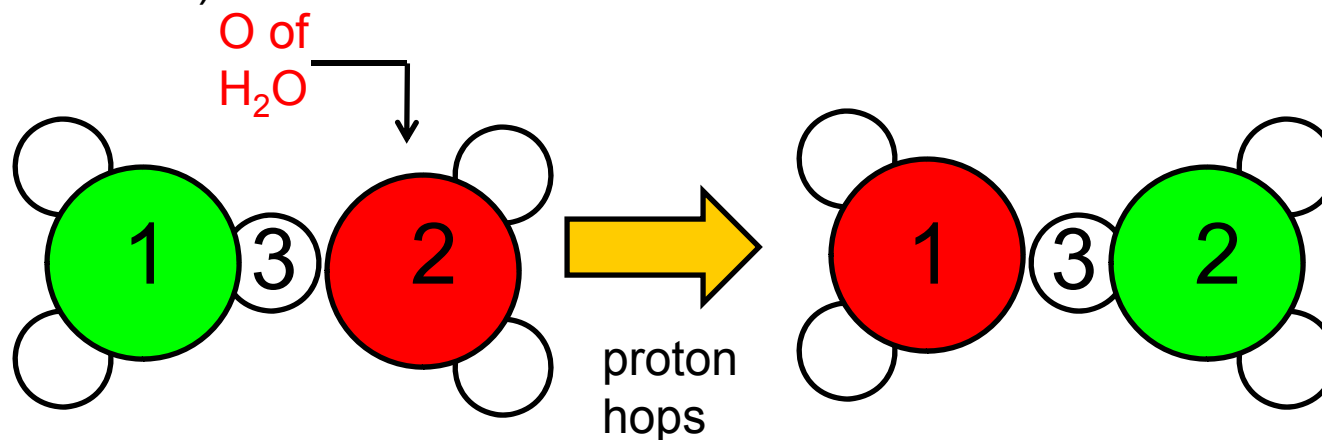
high water ( $\lambda = 22$ )  
large aqueous channels

# Proton Transport – Two Mechanisms

Vehicular diffusion: change in position of center of mass of hydronium ion ( $\text{H}_3\text{O}^+$ )



Structural diffusion (proton shuttling): passing of protons from water molecule to the next (a chemical reaction involving the breaking of a covalent bond)



In bulk water, structural diffusivity is about 70% of total diffusivity.



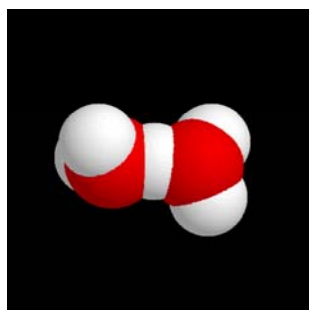
## Proton Transport in Aqueous Systems

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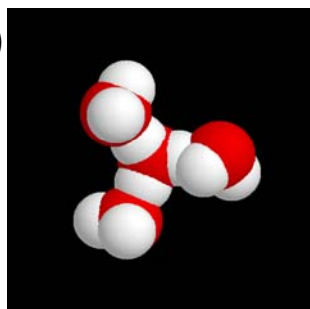
## Understanding of fundamental mechanism of structural diffusion

Hydronium ions exist as hydrated ion complexes like

Zundel ions ( $\text{H}_5\text{O}_2^+$ )



and Eigen ions ( $\text{H}_9\text{O}_4^+$ )



structural diffusion or Proton hopping involves a reaction in which the ground state is likely an Eigen ion and the transition state is a Zundel ion.

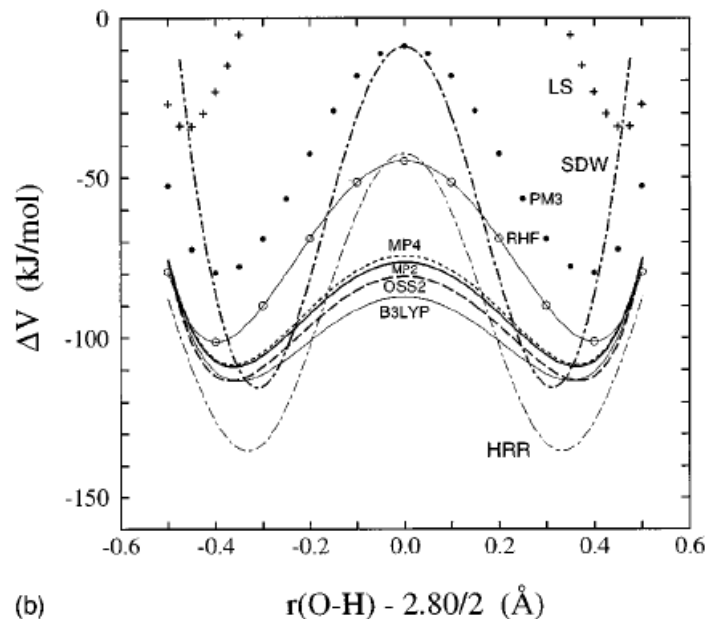
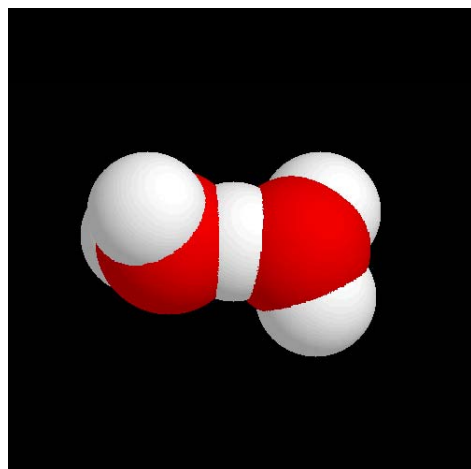


FIG. 3. Proton-transfer energy profiles in the  $\text{H}_5\text{O}_2^+$  ion given by various methods. The distance between the two water molecules was fixed at (a)  $R(\text{O}-\text{O})=2.40 \text{ \AA}$  (the optimized MP2 value at equilibrium) and (b) at  $R=2.80 \text{ \AA}$ . The proton is moved along and on the line connecting the two oxygen atoms. The other geometrical parameters are always fixed at the equilibrium MP2 values. The energy is given relative that of the free oxonium and water monomers. The potentials shown are OSS2 (dashed line), MP2 (solid line), MP4 (short dashes), B3LYP (thin solid line), RHF (thin solid line with open circles), PM3 (filled circles), LS (+), SDW (dash-dot line) and HRR (thin dash-dotted line). Corresponding results for the OSS1(p)–3(p) potentials are given in Fig. 4.

Ojamäe, Shavitt, Singer  
*J. Chem. Phys.* 1998

## Quantum Mechanics

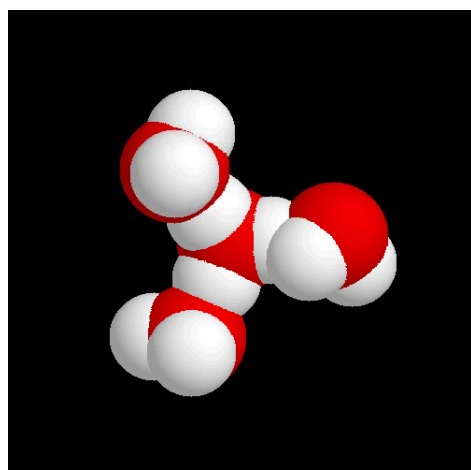
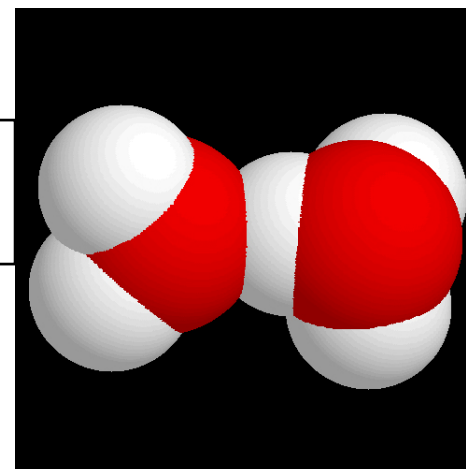
## Molecular Dynamics



Zundel ion

MD at  $\lambda = 4.4$

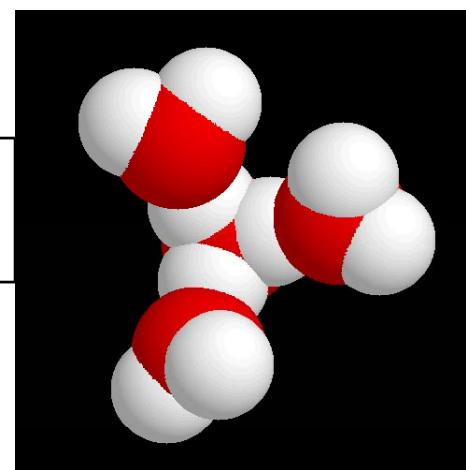
Huang, Braams, Bowman,  
*J. Chem. Phys.* 2005



Eigen ion

MD at  $\lambda = 4.4$

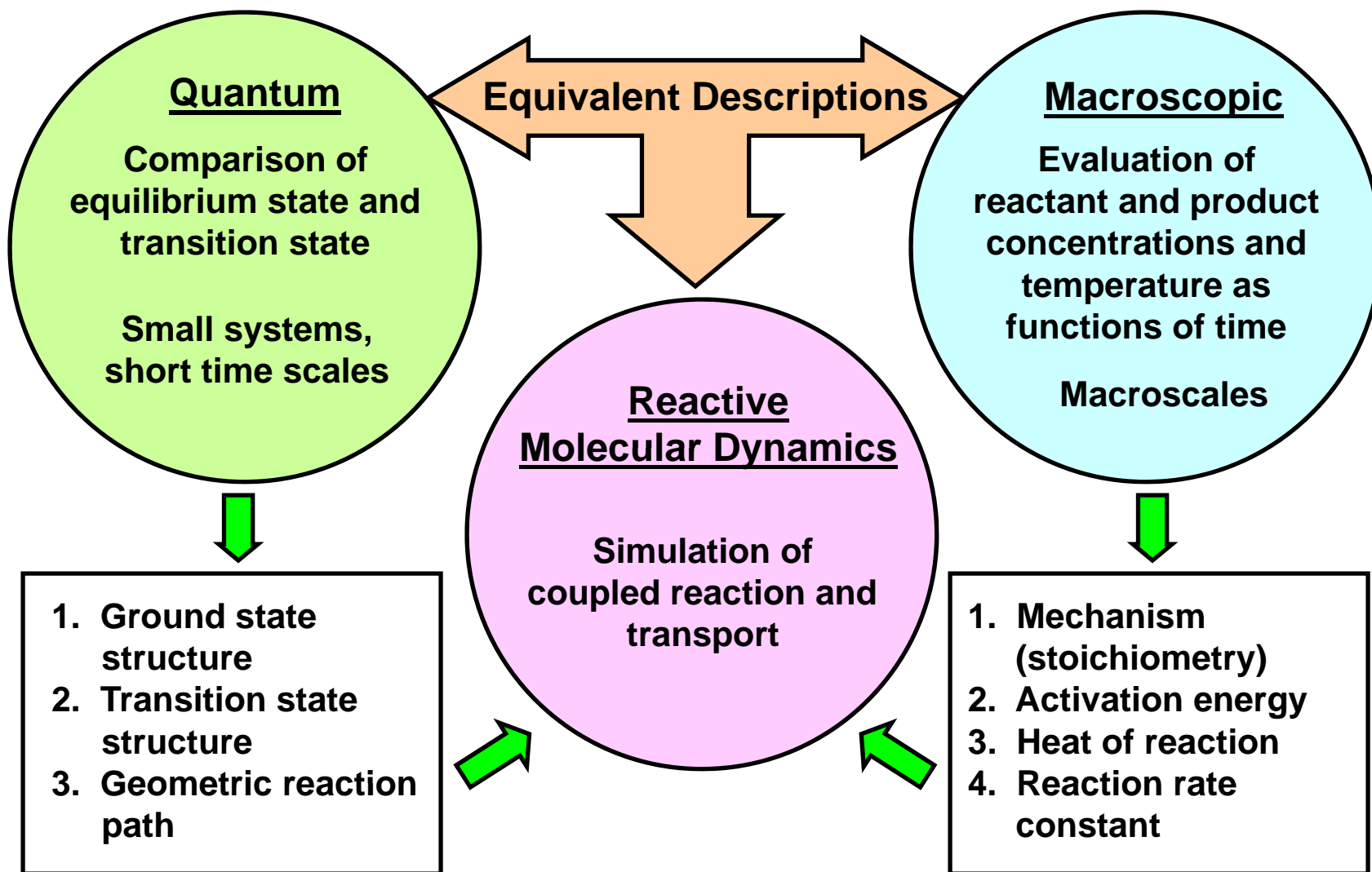
Ojamäe, Shavitt, Singer  
*J. Chem. Phys.* 1998



MD simulations will only approximate the structures from Quantum Mechanics.

## Proton Transport in Aqueous Systems

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# Variety of Approaches of Simulation of Structural Diffusion

Author	Year	Method	Features	System
R. Car & M. Parrinello [1]	1985	Car-Parrinello MD	<ul style="list-style-type: none"> <li>• Computationally expensive</li> <li>• Restricted to small systems</li> </ul>	<ul style="list-style-type: none"> <li>• Excess H<sup>+</sup> in H<sub>2</sub>O [2]</li> <li>• Nonaqueous hydrogen bonded media</li> </ul>
A. Warshel [3]	1980	Empirical Valence Bond	<ul style="list-style-type: none"> <li>• Charge transfer theory of hydrogen bonded complexes</li> <li>• Used to develop MS-EVB, SCI-MS-EVB, MS-EVB3</li> </ul>	<ul style="list-style-type: none"> <li>• Excess H<sup>+</sup> in H<sub>2</sub>O [4,5]</li> <li>• Enzymes</li> </ul>
R.G. Schmidt & J. Brickmann [6]	1997	Mixed MD and MC	<ul style="list-style-type: none"> <li>• Proton hopping between titratable sites</li> <li>• Criteria - Distance between donor and acceptor</li> </ul>	<ul style="list-style-type: none"> <li>• Excess H<sup>+</sup> in H<sub>2</sub>O</li> <li>• Proton in amino acid</li> </ul>
M.A. Lill & V. Helms [7]	2001	Q-HOP MD	<ul style="list-style-type: none"> <li>• Proton hopping between titratable sites</li> <li>• Criteria - Distance and environmental effect of the surrounding group</li> </ul>	<ul style="list-style-type: none"> <li>• Excess H<sup>+</sup> in H<sub>2</sub>O</li> <li>• Aspartic acid in H<sub>2</sub>O</li> <li>• Imidazole ring in H<sub>2</sub>O</li> </ul>

[1] R. Car and M. Parrinello, *Phys.Rev.Lett.*, 55, 2471 (1985).

[2] M. Tuckerman, et al., *J.Chem.Phys.*, 103, 150 (1995).

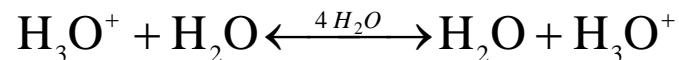
[3] A. Warshel and R.M. Weiss, *J. Am. Chem. Soc.*, 102, 6218 (1980).

[4] J. Lobaugh and G.A. Voth, *J. Chem. Phys.*, 104, 2056 (1996).

[5] D.E. Sagnella and M.E. Tuckerman, *J. Chem. Phys.*, 108, 2073 (1998).

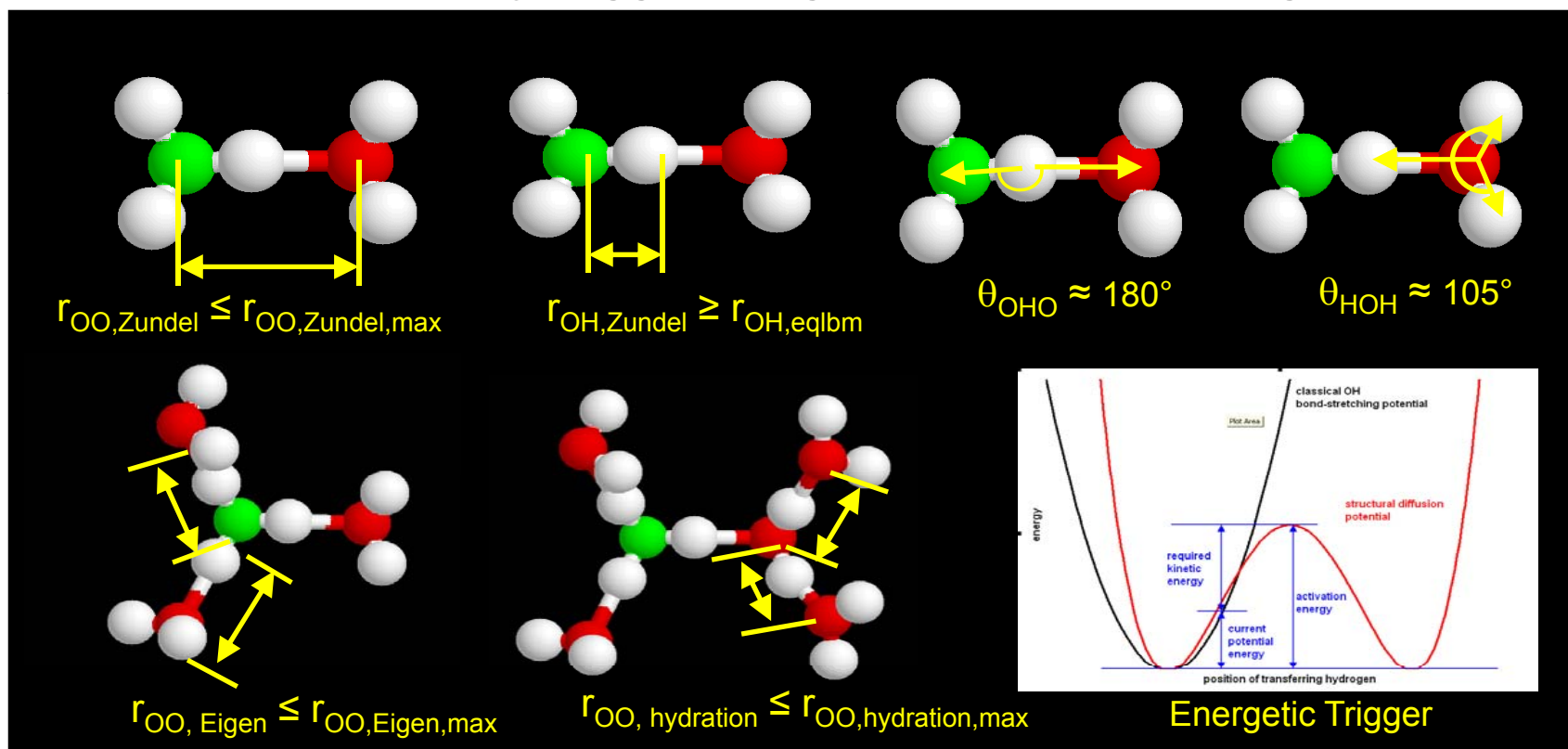
[6] R.G. Schmidt and J. Brickmann., *Ber. Bunsenges. Phys. Chem.*, 101, 1816 (1997).

[7] M.A. Lill and V. Helms, *J. Chem. Phys.*, 115, 7993 (2001).



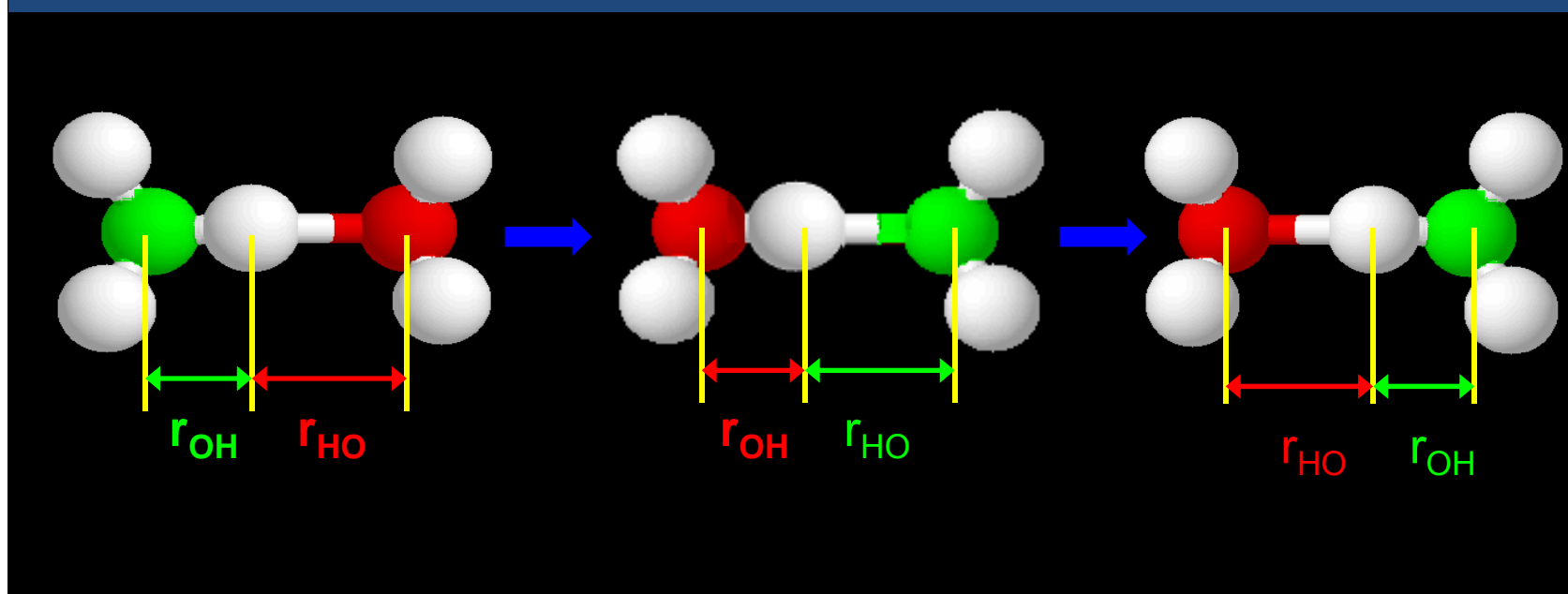
At each step of conventional MD simulation, check if reactant ( $\text{H}_3\text{O}^+$ ) is in a reactive configuration.

## Step 1. Satisfy triggers (6 geometric and 1 energetic)



### Step 2. Instantaneous Reaction

O of  $\text{H}_3\text{O}^+$  = green  
O of  $\text{H}_2\text{O}$  = red  
H = white

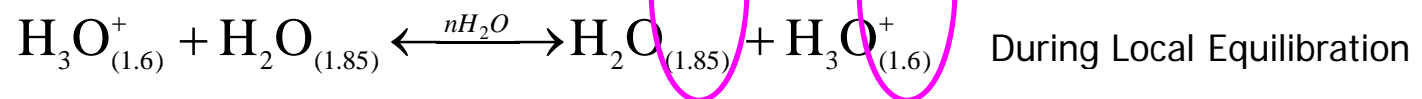
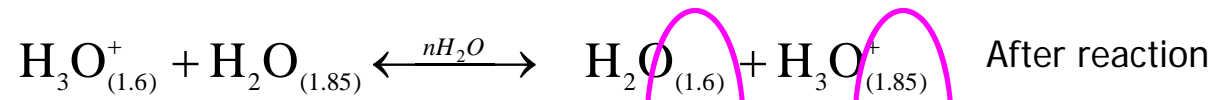


- Exchange identities of  $\text{H}_3\text{O}^+$  and  $\text{H}_2\text{O}$  molecules
- Move proton over to the newly formed hydronium ion so that  $r_{\text{OH}}$  of the hydronium before and after reaction are the same



## Step 3. Local Equilibration

- There is an increase in the potential energy of the system and disturbance of structure



- Helps in restoring system structurally and maintaining the correct heat of reaction

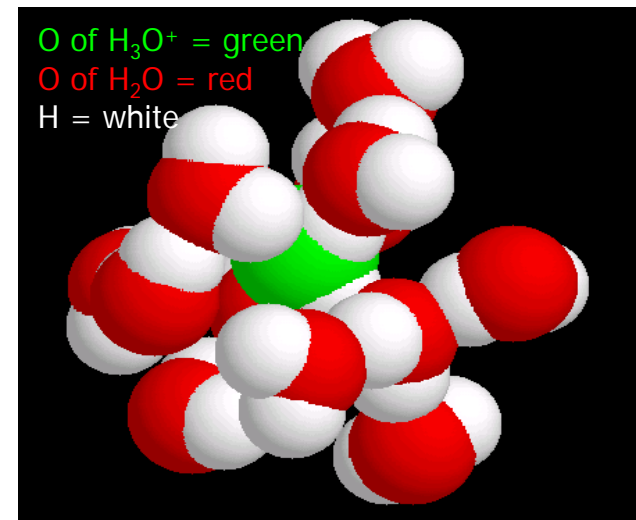
## Objective Function

$$F_{obj} = w_1 \Delta U^{\text{RMS}} + w_2 \Delta g(r)^{\text{RMS}}$$

energetic term                      structural term

$$\Delta U^{\text{RMS}} = \sqrt{\left( \frac{U_{\text{After}} - U_{\text{Before}}}{U_{\text{Before}}} \right)^2}$$

$$\Delta g(r)^{\text{RMS}} = \sqrt{\frac{1}{N_{\text{pairs}}} \sum_{i=1}^{i=N_{\text{pairs}}} \left( \frac{r_{ij} - r_{ij}^{\text{target}}}{r_{ij}^{\text{target}}} \right)^2}$$



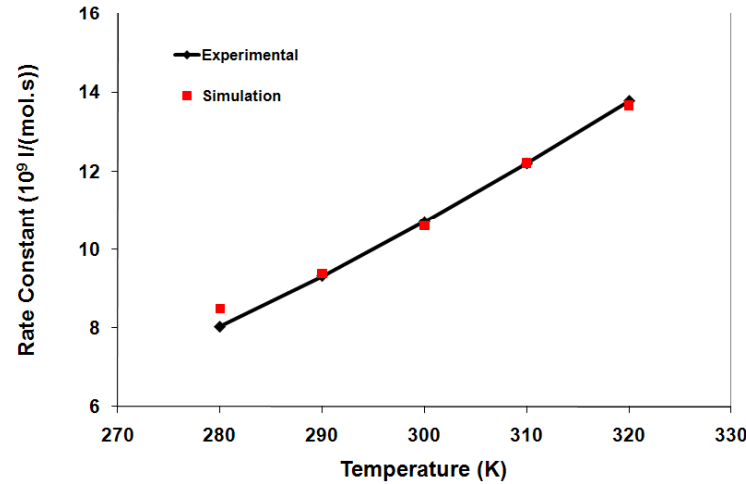
Snapshot representing the complex hydrogen bonding network

# Proton Transport in Bulk Water

reaction:  $\text{H}_3\text{O}^+ + \text{H}_2\text{O} \rightarrow \text{H}_2\text{O} + \text{H}_3\text{O}^+$   
 rate law:  $\text{rate} = k [\text{H}_3\text{O}^+][\text{H}_2\text{O}]$

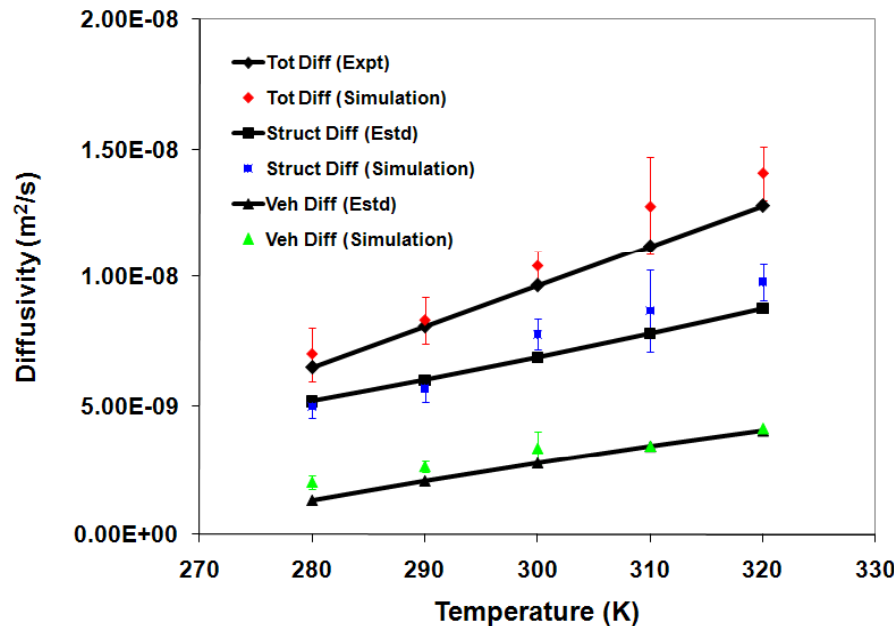
$$k = k_o \exp\left(-\frac{E_a}{RT}\right)$$

- Adjust triggers to fit experimental rate.
- Predict transport properties.



experimental data from  
 Luz, Z.; Meiboom, S. *J. Am. Chem. Soc.*, 1964.

**RMD rate constant within 6% of experiment.**

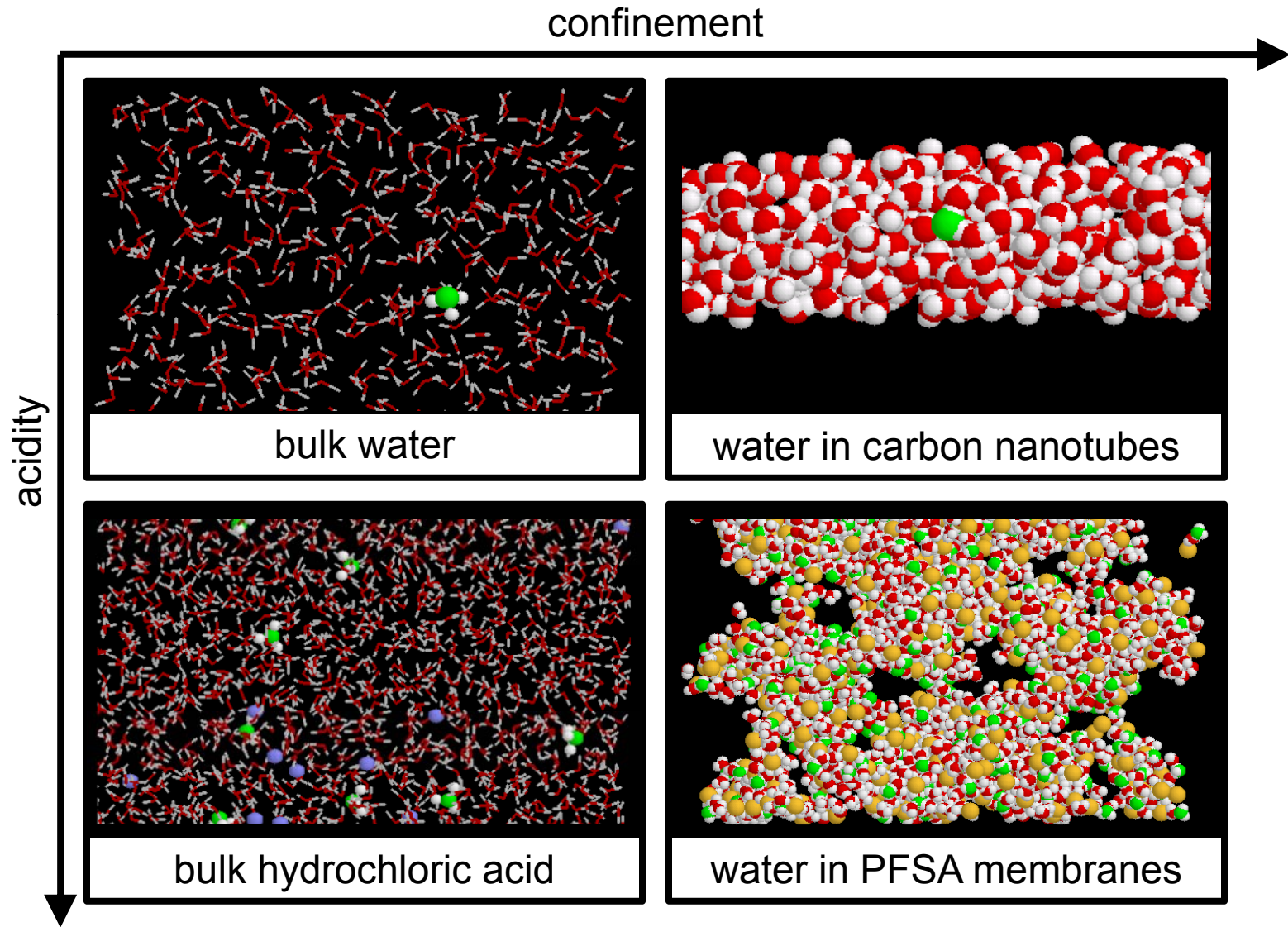


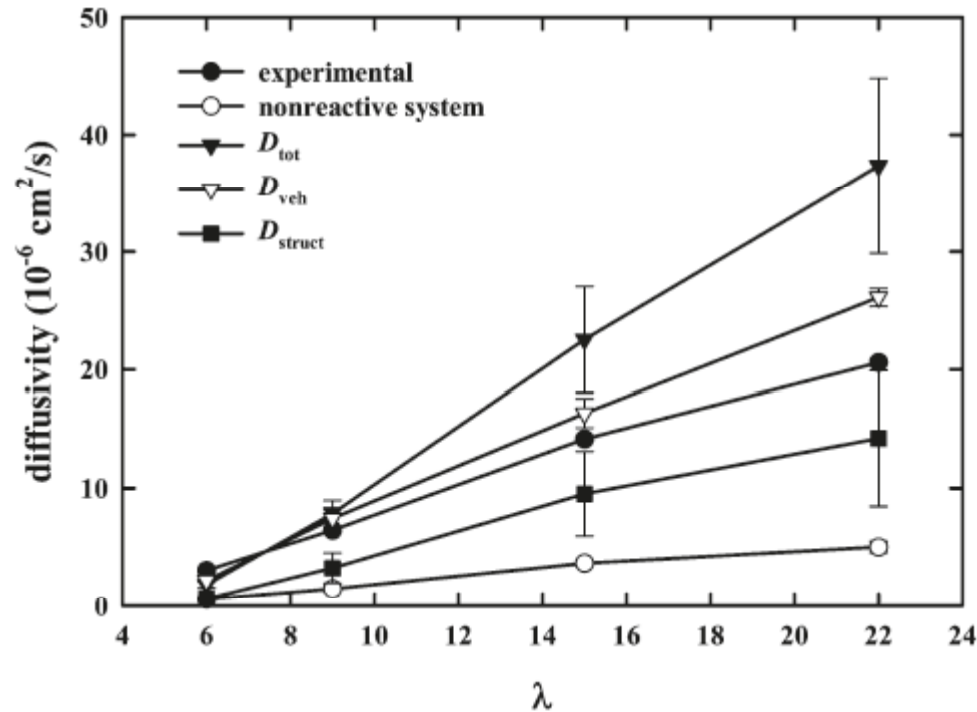
Charge self-diffusivity prediction

- semi-quantitative agreement with experiment
- decomposition into structural and vehicular components
- structural is 60-70% of total
- correct temperature dependence
- structural and vehicular components are uncorrelated

$$D_{tot} = \lim_{\tau \rightarrow \infty} \frac{\langle \Delta \vec{r}_{veh}^2 \rangle + \langle \Delta \vec{r}_{struct}^2 \rangle + 2 \langle \Delta \vec{r}_{veh} \Delta \vec{r}_{struct} \rangle}{2d\tau}$$

# Acidity and Confinement Effects on Proton Mobility



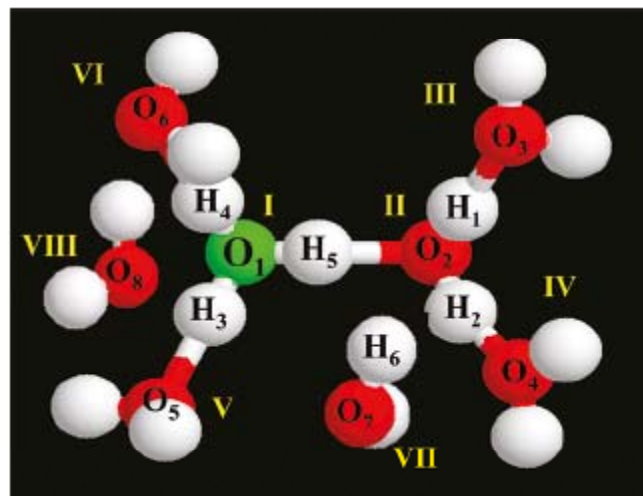


Introducing structural diffusion into the simulation via the same RMD algorithm that was used for bulk water, HCl solutions and water in carbon nanotubes

- provided a correct quantitative trend
- but the total charge diffusivity was too large
- the vehicular component significantly increased relative to nonreactive MD

The presence of reaction disturbs the local hydrogen-bonding network, resulting in higher mobility of protons (and water (not shown)).

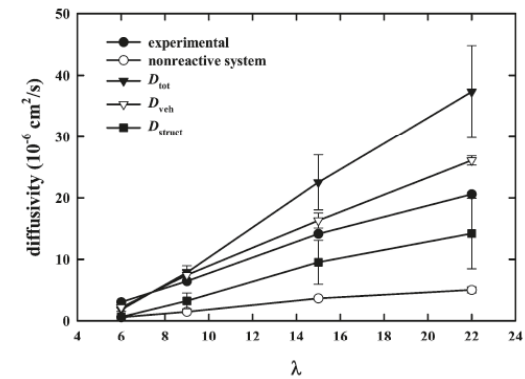
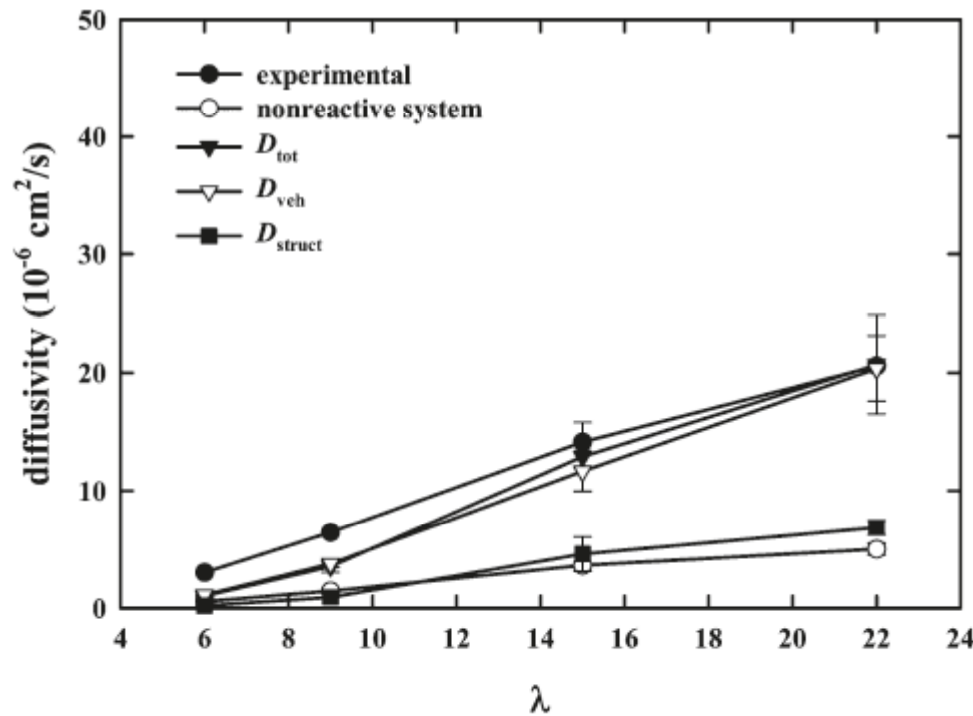
## Method 2: Attempt to better maintain hydrogen-bonding network after reaction



**Figure 1.** Typical structure of the reactants surrounded by six hydrating molecules in the bulk water system. Atom and molecule labels serve for identification purposes in Table 2. O of  $\text{H}_3\text{O}^+$ , green; O of  $\text{H}_2\text{O}$ , red; H, white.

Include more water molecules in local equilibration after instantaneous reaction.

# Water and Proton Transport in Nafion (Method 2)



Method 1

Introducing a more stable hydrogen-bonding network after reaction

- provided a correct quantitative trend
- significantly improved quantitative agreement.
- the vehicular component now similar to nonreactive MD

Still observed higher water diffusivity.

$$D_{\text{tot}} = \lim_{\tau \rightarrow \infty} \frac{\langle \Delta \vec{r}_{\text{veh}}^2 \rangle + \langle \Delta \vec{r}_{\text{struct}}^2 \rangle + 2\langle \Delta \vec{r}_{\text{veh}} \Delta \vec{r}_{\text{struct}} \rangle}{2d\tau} \quad (8)$$

The above definition can be decomposed as

$$D_{\text{veh}} \equiv \lim_{\tau \rightarrow \infty} \frac{\langle \Delta \vec{r}_{\text{veh}}^2 \rangle}{2d\tau} \quad (9a)$$

$$D_{\text{struct}} \equiv \lim_{\tau \rightarrow \infty} \frac{\langle \Delta \vec{r}_{\text{struct}}^2 \rangle}{2d\tau} \quad (9b)$$

$$D_{\text{corr}} \equiv \lim_{\tau \rightarrow \infty} \frac{2\langle \Delta \vec{r}_{\text{veh}} \Delta \vec{r}_{\text{struct}} \rangle}{2d\tau} \quad (9c)$$

The correlation term is zero in bulk water, HCl solutions and in carbon nanotubes.

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## morphology of bulk hydrated membrane

Nafion

EW = 1144

$\lambda = 6 \text{ H}_2\text{O}/\text{HSO}_3$

T = 300 K

Snapshots of  
the aqueous  
nanophase  
show a tortuous  
path.

legend:

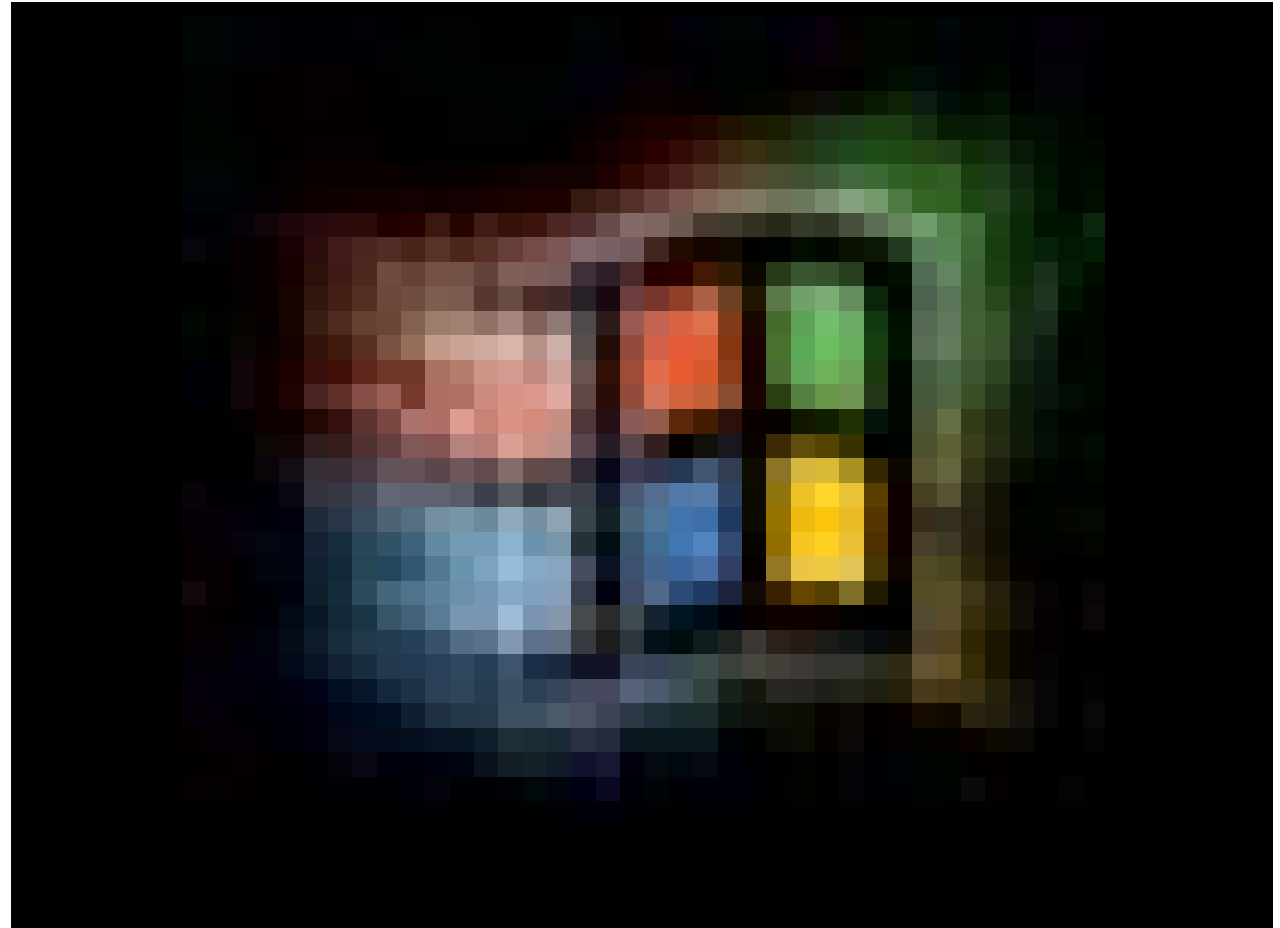
O of  $\text{H}_2\text{O}$  = red

H = white

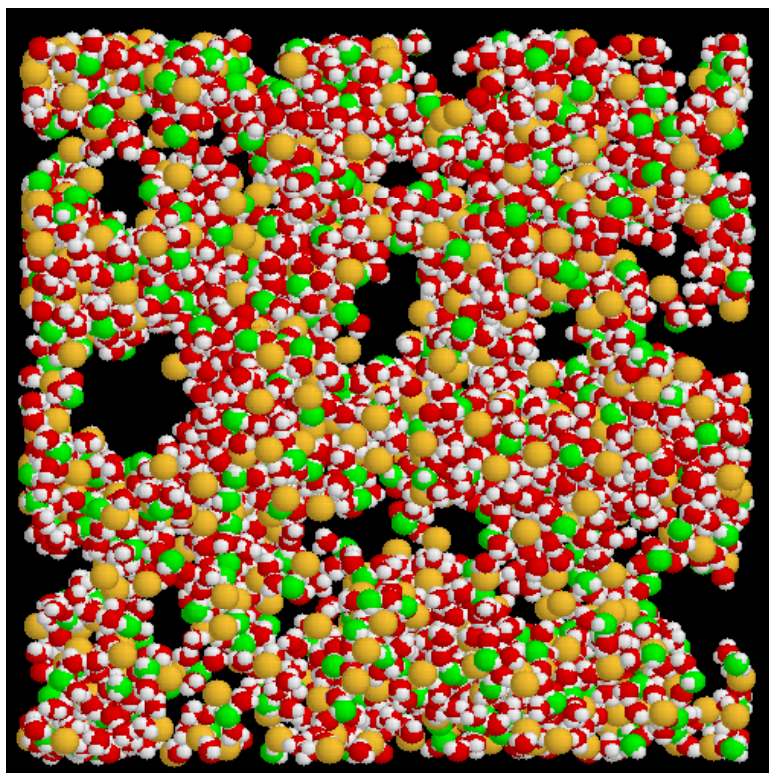
O of  $\text{H}_3\text{O}^+$  = green

S = orange

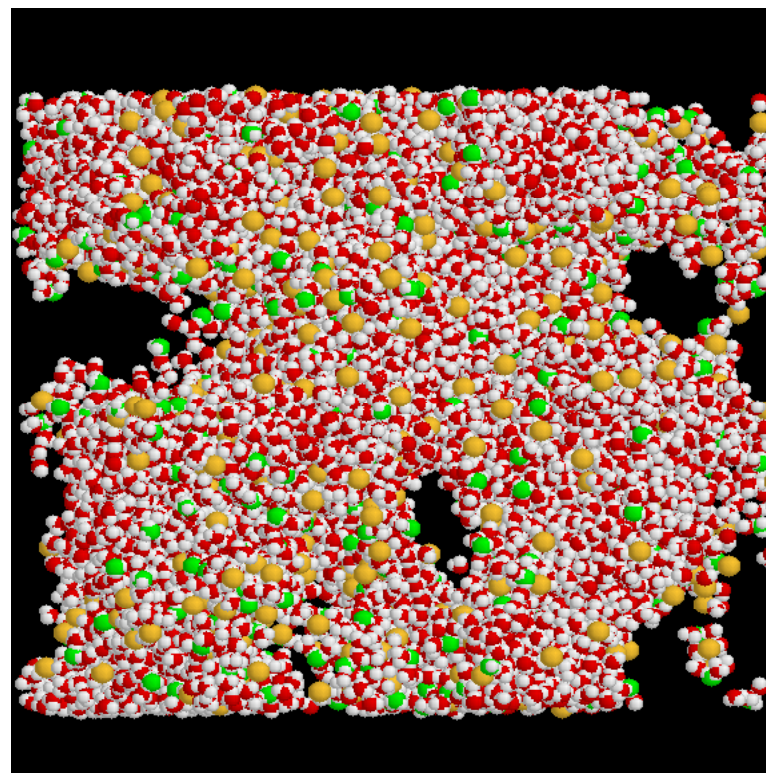
remainder of polymer electrolyte not shown



## PEM morphology is a function of water content



Nafion (EW = 1144)  $\lambda = 6$  H<sub>2</sub>O/HSO<sub>3</sub>  
small aqueous channels



Nafion (EW = 1144)  $\lambda = 22$  H<sub>2</sub>O/HSO<sub>3</sub>  
much larger aqueous channels

As the membrane becomes better hydrated, the channels in the aqueous domain become larger and better connected, resulting in higher conductivity.  
(The challenge to finding high-temperature membranes is to find one that can retain moisture at elevated temperatures.)

Einstein Relation – long time slope of mean square displacement to observation time

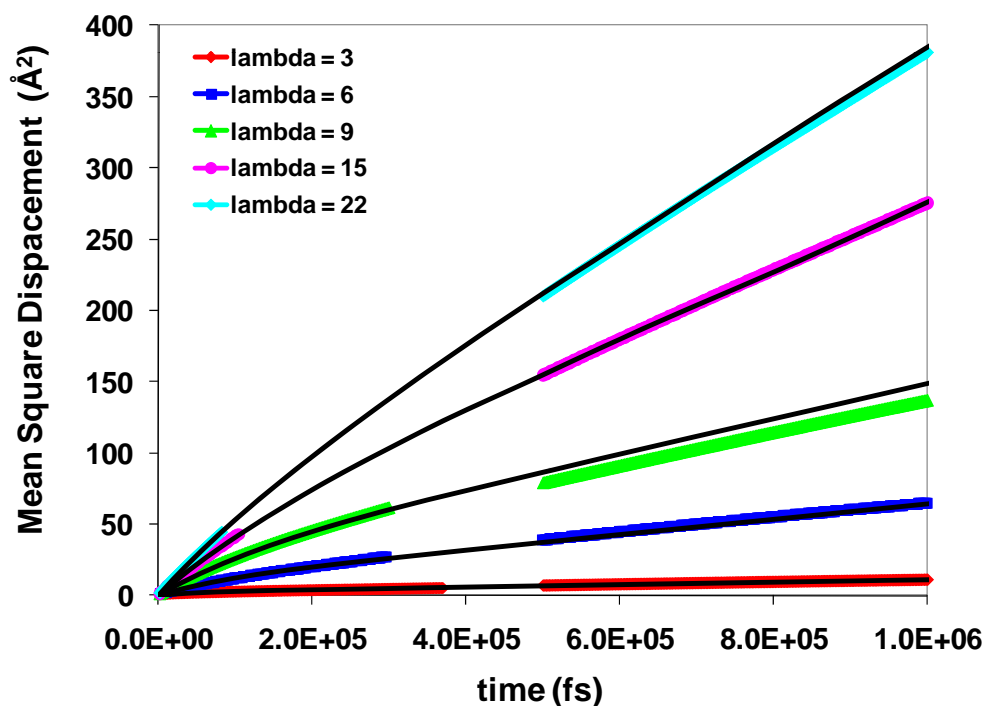
$$D = \lim_{\tau \rightarrow \infty} \frac{MSD}{2d\tau} = \lim_{\tau \rightarrow \infty} \frac{\langle [r_i(t + \tau) - r_i(t)]^2 \rangle}{2d\tau}$$

position of particle  $i$  at time  $t$

Einstein Relation works well for bulk systems.

But for simulation in PEMs, we can't reach the long-time limit required by Einstein relation.

MD simulations alone are not long enough.

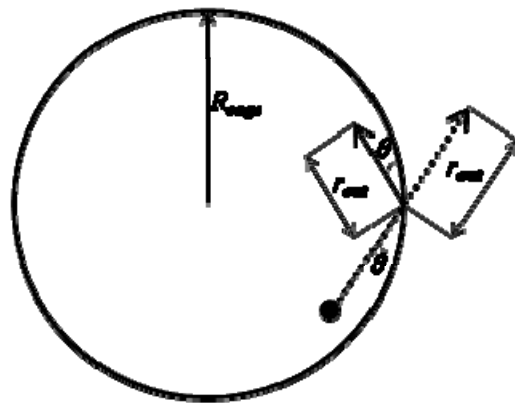


Liu, J. et al. *J. Phys. Chem. C* 2010.

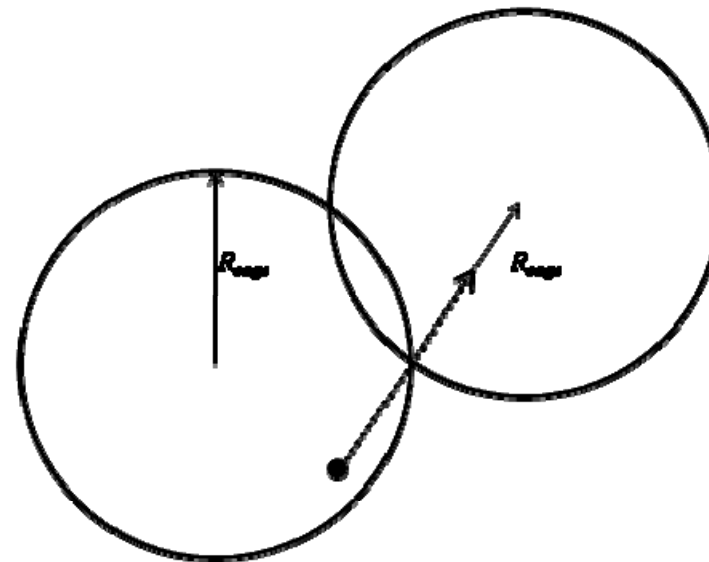
MSDs don't reach the long-time (linear) regime.

## Mesoscale Model

- non-interacting point particles (no energies, no forces)
- sample velocities from a Maxwell-Boltzmann distribution
- two parameters
  - cage size
  - cage-to-cage hopping probability
- parameters fit to MSD from Molecular Dynamics Simulation
- runs on a laptop in a few minutes

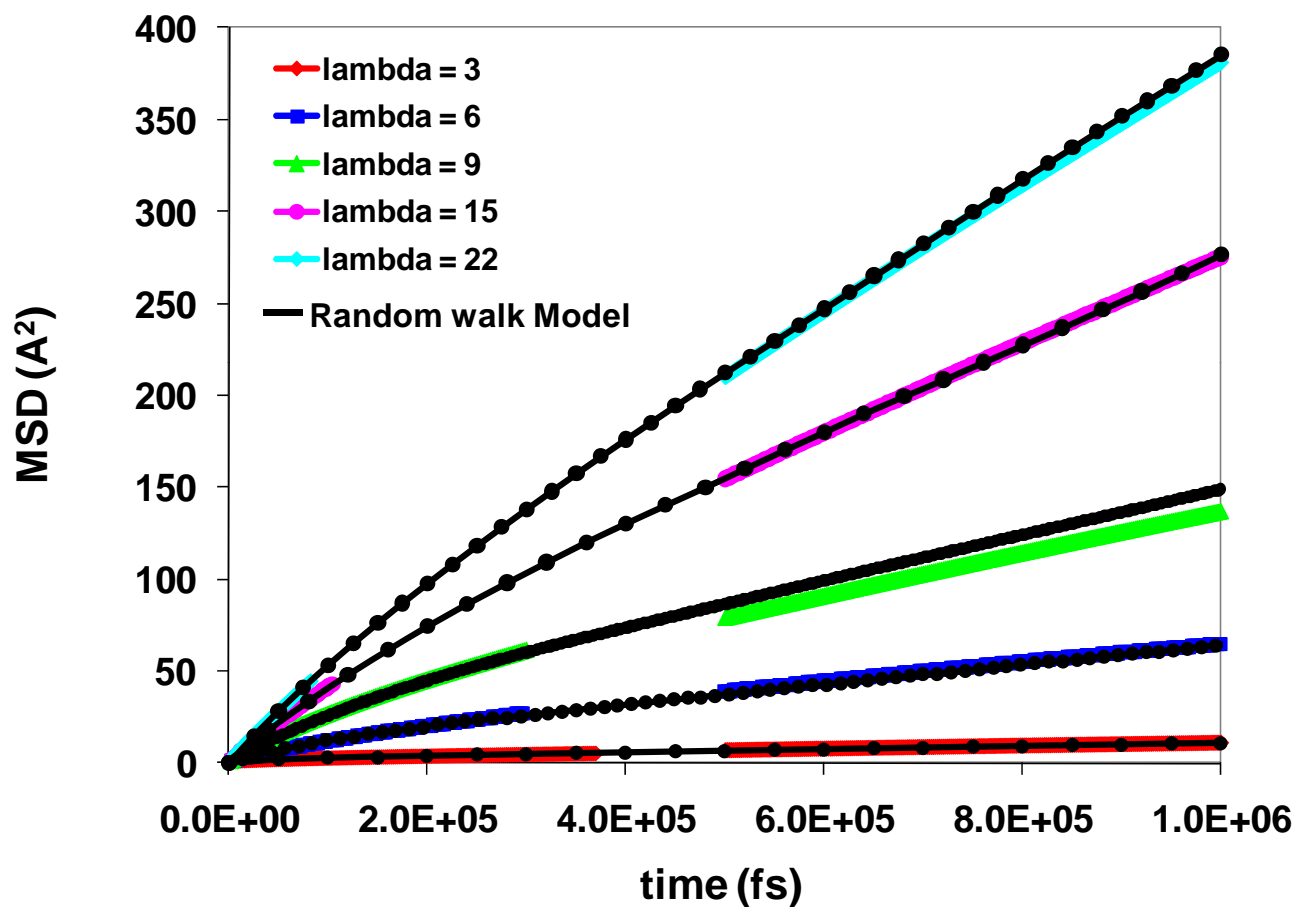


unsuccessful move



successful move

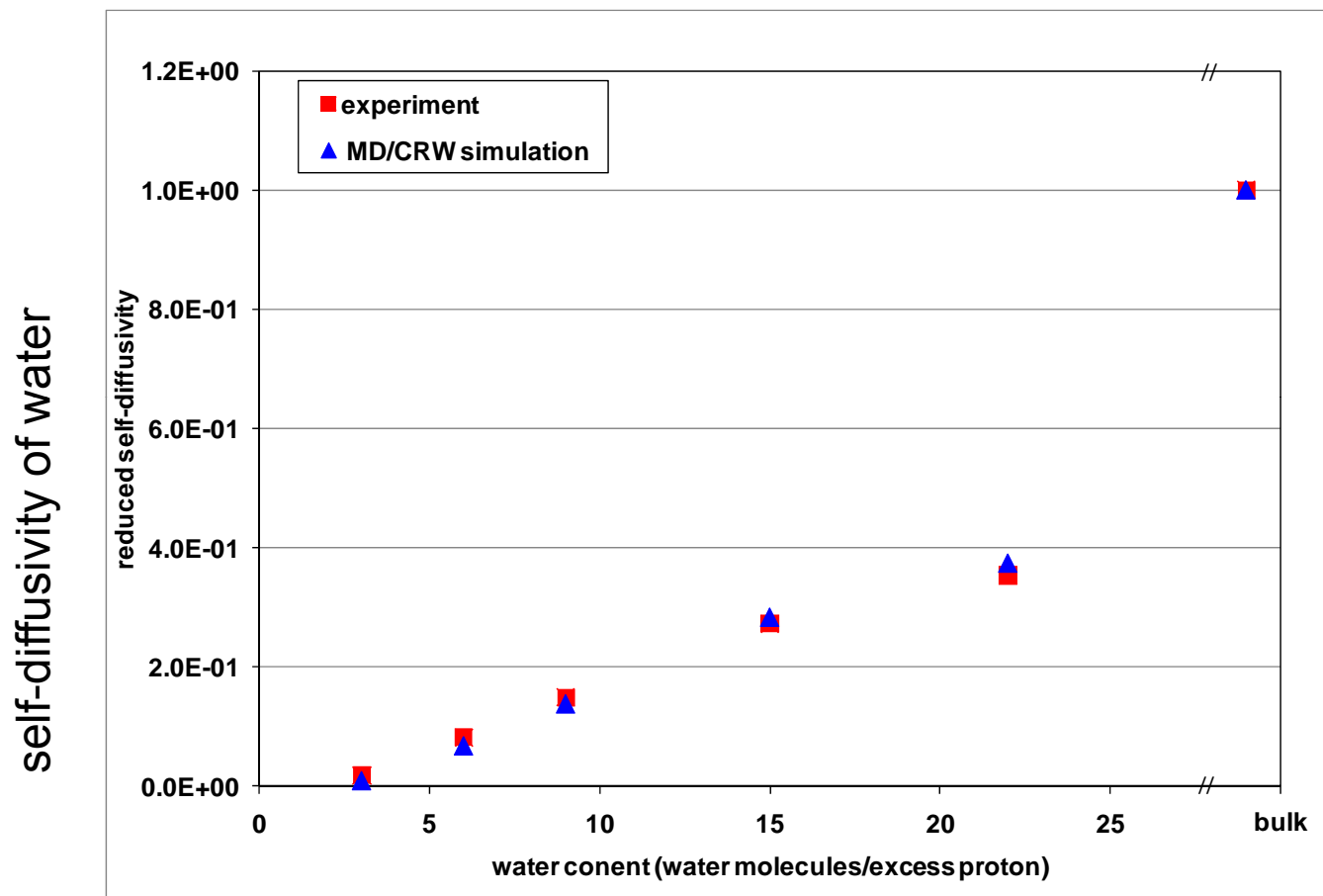
Calvo-Muñoz, E.M., Esai Selvan, M., Xiong, R., Ojha, M., Keffer, D.J., Nicholson, D.M., Egami, T", *Phys. Rev. E*, **83**(1) 2011 article # 011120.



Calvo-Muñoz, E.M., Esai Selvan, M., Xiong, R., Ojha, M., Keffer, D.J., Nicholson, D.M., Egami, T., *Phys. Rev. E*, **83**(1) 2011 article # 011120.

- Fit MD results (1 ns) to Confined Random Walk (CRW) Theory.
- Extend Mean Square Displacement to long-time limit (100 ns).
- Extract water diffusivity.

## Comparison of MD/CRW Simulation with Experiment



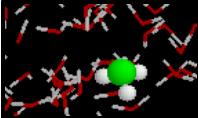
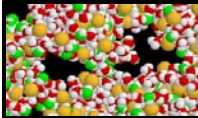
Robison, R. A.; Stokes, R. H. *Electrolyte Solutions*, 1959.  
Nafion (EW=1100,) Kreuer, K. D. *Solid State Ionics* **1997**.  
Esai Selvan, M., Calvo-Muñoz, E.M., Keffer, D.J., *J. Phys. Chem. B*, dx.doi.org/10.1021/jp1115004, 2011.

- Excellent agreement between simulation and experiment for water diffusivity as a function of water content
- Can we predict the self-diffusivity of water without computationally expensive simulations?

## Proton Transport in Aqueous Systems

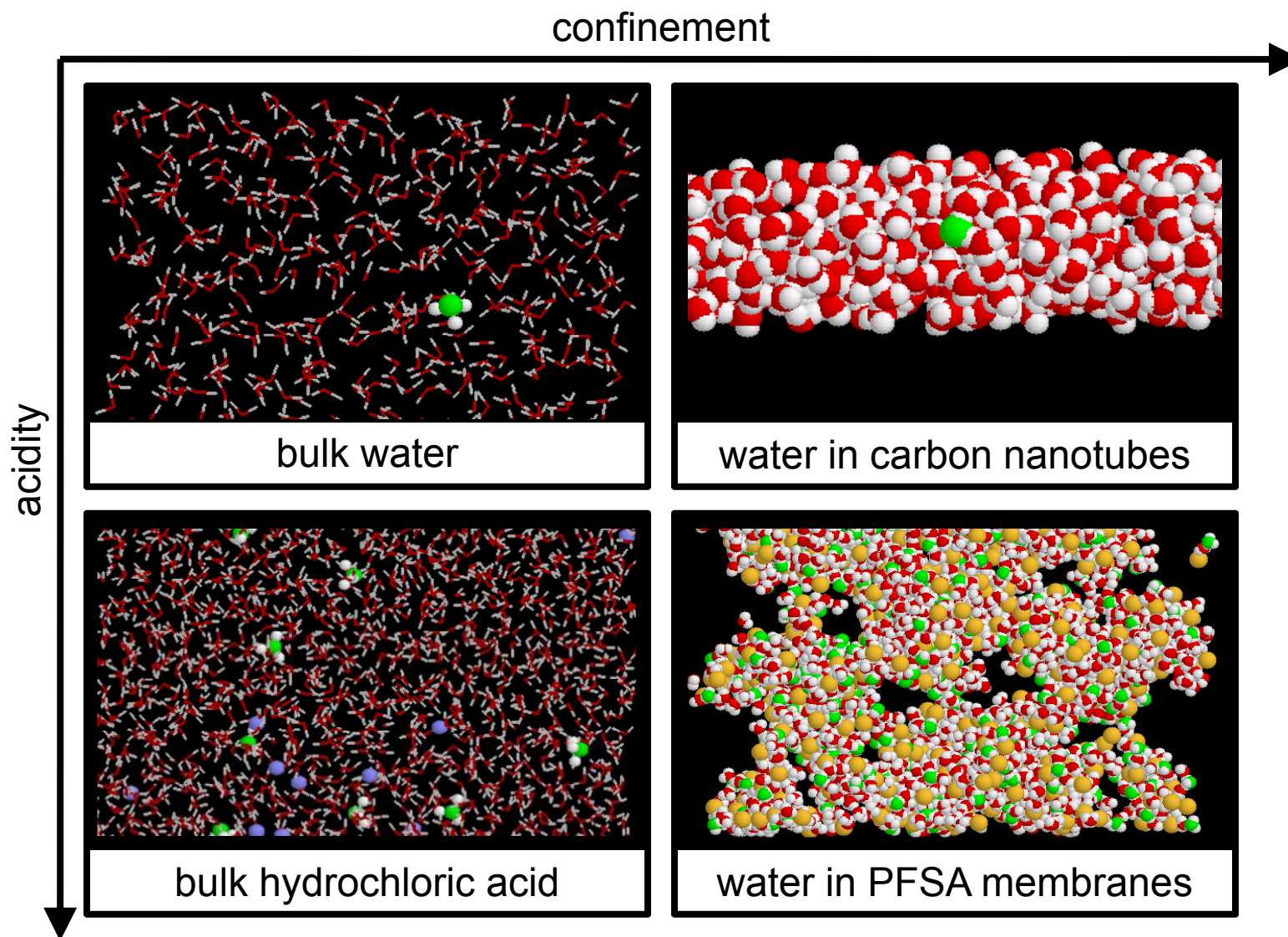
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  - II.D. Percolation Theory
- III. Conclusions

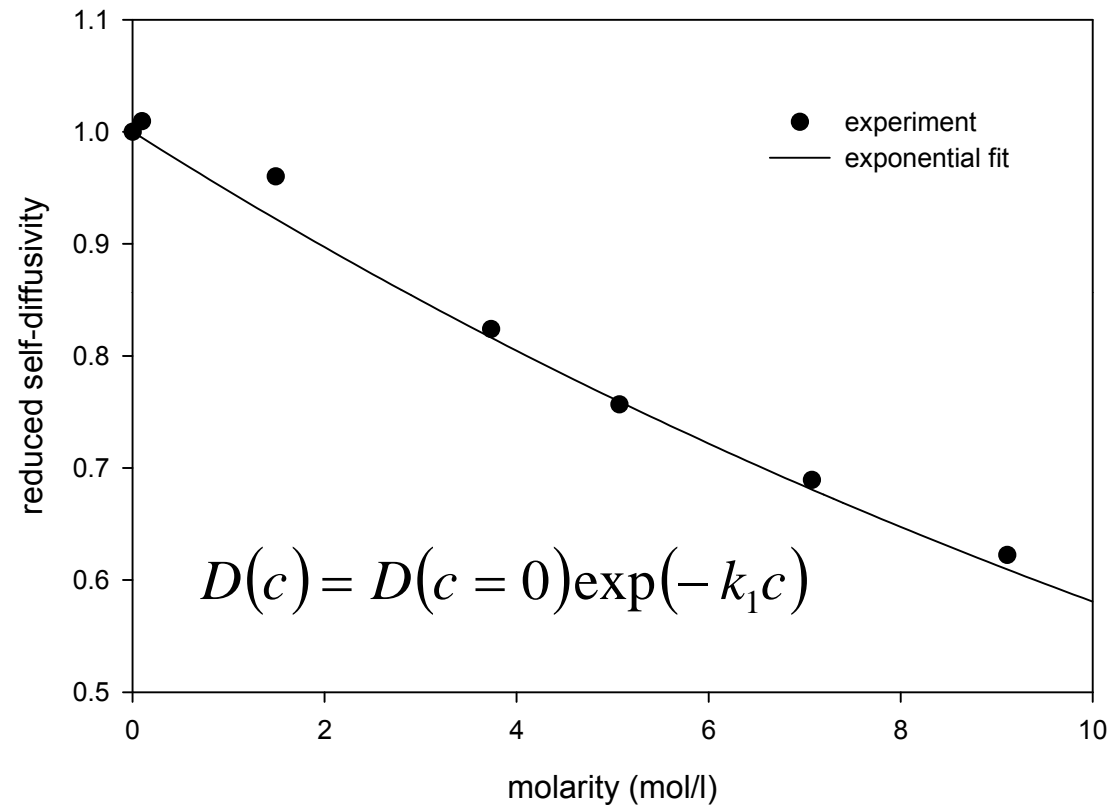
# Three Factors: Acidity, Confinement & Connectivity

	bulk water	water in PFSA membranes (Nafion EW=1144)
acidity	 <ul style="list-style-type: none"><li>• <math>\text{H}_3\text{O}^+</math> concentration is dilute</li><li>• <math>\lambda = 5.6 \cdot 10^8 \text{ H}_2\text{O}/\text{H}^+</math> (pH=7)</li></ul>	 <ul style="list-style-type: none"><li>• <math>\text{H}_3\text{O}^+</math> concentration</li><li>• <math>\lambda = 3 \text{ H}_2\text{O}/\text{HSO}_3</math>, pH <math>\approx -0.59</math> (minimally hydrated)</li><li>• <math>\lambda = 22</math>, pH <math>\approx -0.22</math> (saturated)</li></ul>
confinement	<ul style="list-style-type: none"><li>• interfacial surface area is zero</li></ul>	<p>interfacial surface area</p> <ul style="list-style-type: none"><li>• <math>163 \text{ \AA}^2/\text{H}_2\text{O}</math> or <math>2460 \text{ m}^2/\text{g}</math> (<math>\lambda = 3</math>)</li><li>• <math>23 \text{ \AA}^2/\text{H}_2\text{O}</math> or <math>1950 \text{ m}^2/\text{g}</math> (<math>\lambda = 22</math>)</li></ul>
connectivity	<ul style="list-style-type: none"><li>• no connectivity issues</li></ul>	<ul style="list-style-type: none"><li>• connectivity of aqueous domain deteriorates as water content decreases</li></ul>



# Acidity and Confinement Effects on Proton Mobility

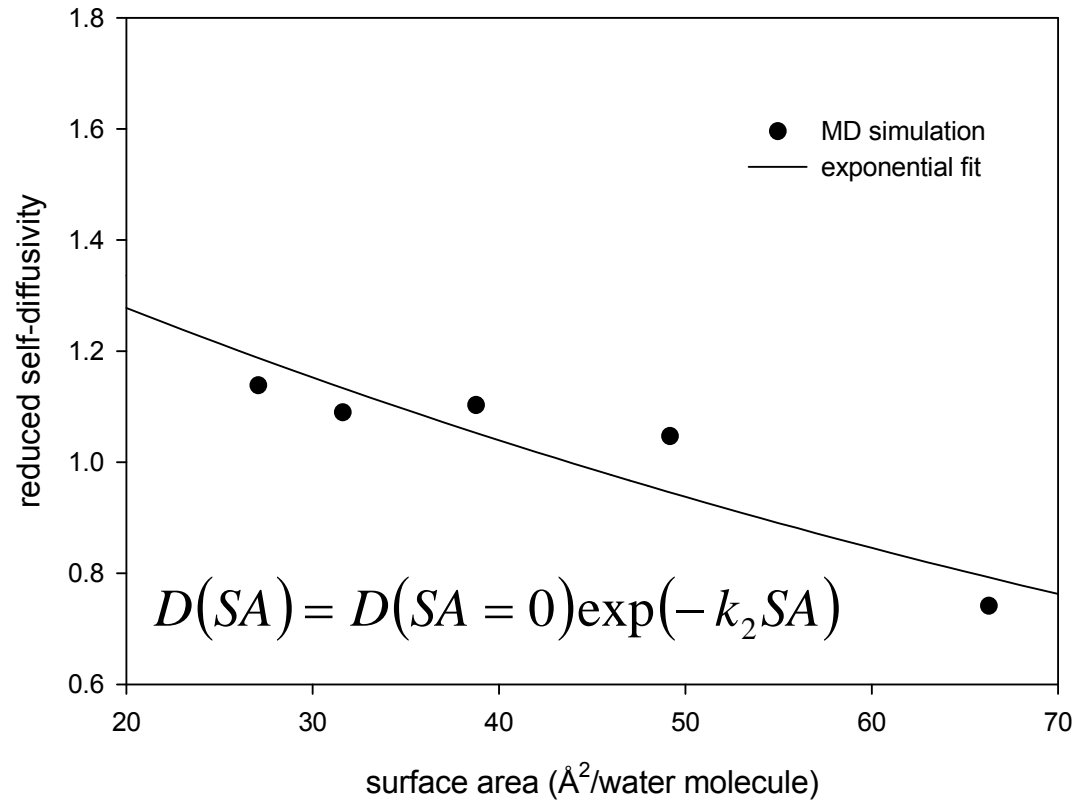




Dippel, T.; Kreuer, K. D. *Solid State Ionics* **1991**, *46*, 3-9.

- In bulk systems, the diffusivity of water decreases as the concentration of HCl increases.
- The behavior is well fit by an exponential fit.

# Water Mobility in Nanotubes – Effect of Confinement



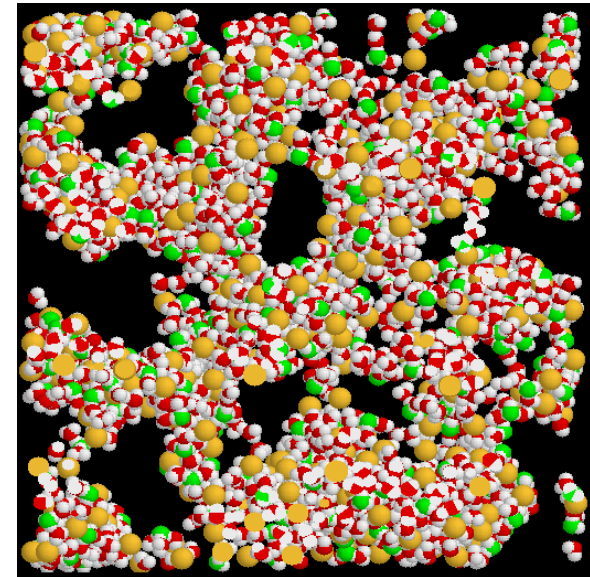
Esai Selvan, M.; Keffer, D. J.; Cui, S.; Paddison, S. J. *Molec. Sim.* **2010**.

- In carbon nanotubes, the diffusivity of water decreases as the radius of the nanotube decreases.
- The behavior is fit by an exponential fit.

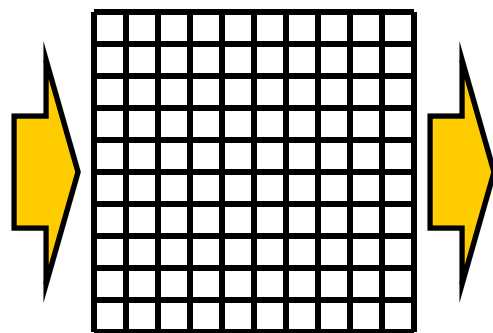
Invoke Percolation Theory to account for connectivity of aqueous domain within PEM and obtain effective diffusivity.

$$\int_0^{\infty} \frac{D_{eff} - D}{\left(\frac{z}{2} - 1\right) D_{eff} + D} g(D) dD = 0$$

$$g(D) = p_{EMA} \delta(D - D_b) + (1 - p_{EMA}) \delta(D - D_o)$$

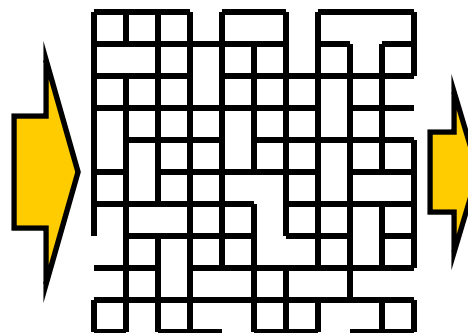


Percolation theory relates the effective diffusivity to the fraction of bonds that are blocked to diffusion.



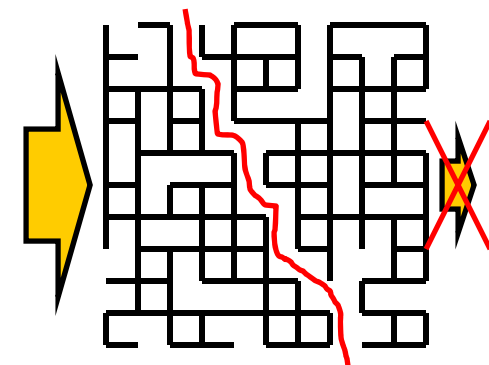
no blocked bonds

$$D = D_{open}$$



some blocked bonds

$$0 < D < D_{open}$$

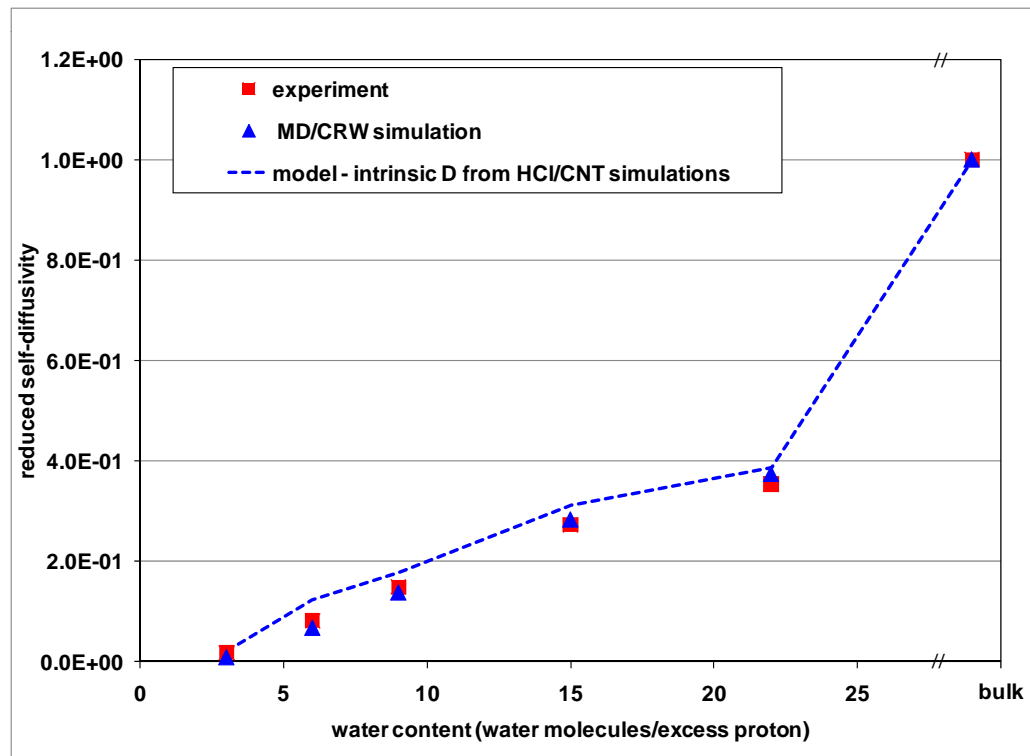


beyond threshold

$$D = 0$$

- Acidity – characterized by concentration of  $\text{H}_3\text{O}^+$  in aqueous domain (exponential fit of HCl data)
- Confinement – characterized by interfacial surface area (exponential fit of carbon nanotube data)
- Connectivity – characterized by percolation theory (fit theory to MD/CRW water diffusivity in PEMs)

Esai Selvan, M., Calvo-Muñoz, E.M., Keffer, D.J., J. Phys. Chem. B 115(12) 2011 pp 3052–3061.



Excellent agreement of theory with both simulation and experiment.

Theory uses only structural information to predict transport property.

Water is solved!  
What about charge transport?

## What about Proton Transport?

We have shown thus far that we can model the transport of **water** fairly accurately using either

1. detailed MD/CRW simulation (months on a supercomputer)
2. analytical model based on acidity, confinement & connectivity (minutes on a laptop computer)

We now want to repeat this process for **protons**. After all, it is the transport of protons that completes the electrical circuit in a fuel cell.

Why did we start with water?

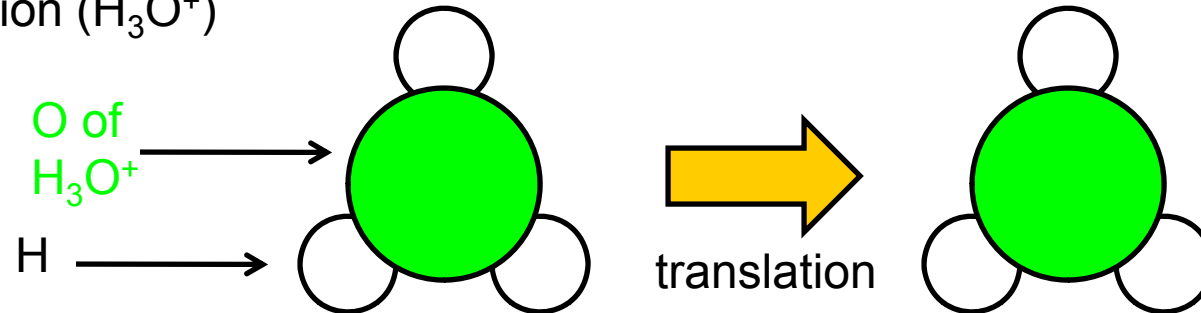
Diffusion of water is easier to describe.

Water is transported only via vehicular diffusion (changes in the center of mass of the water molecules).

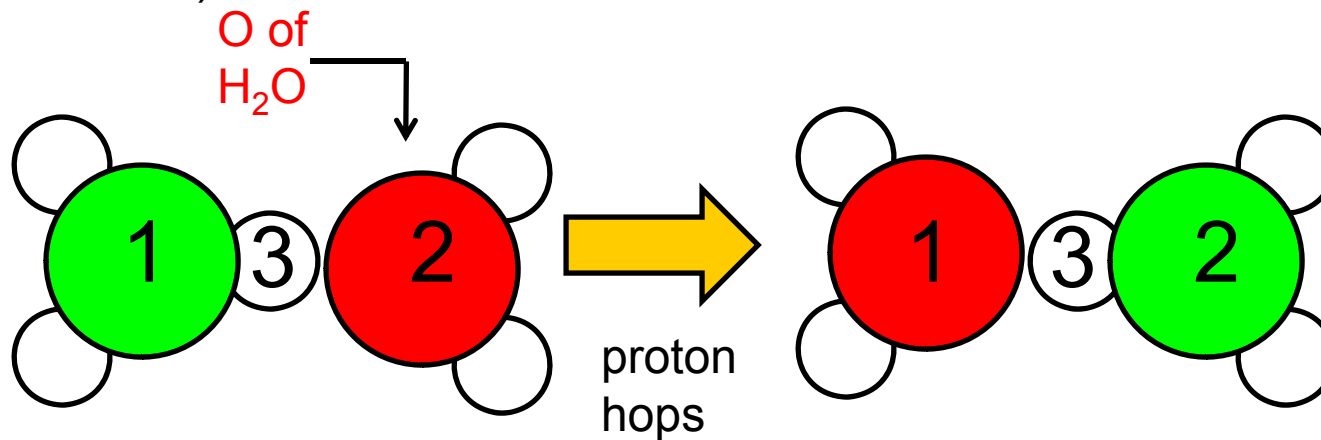
There are two mechanisms for proton transport.

# Proton Transport – Two Mechanisms

Vehicular diffusion: change in position of center of mass of hydronium ion ( $\text{H}_3\text{O}^+$ )



Structural diffusion (proton shuttling): passing of protons from water molecule to the next (a chemical reaction involving the breaking of a covalent bond)



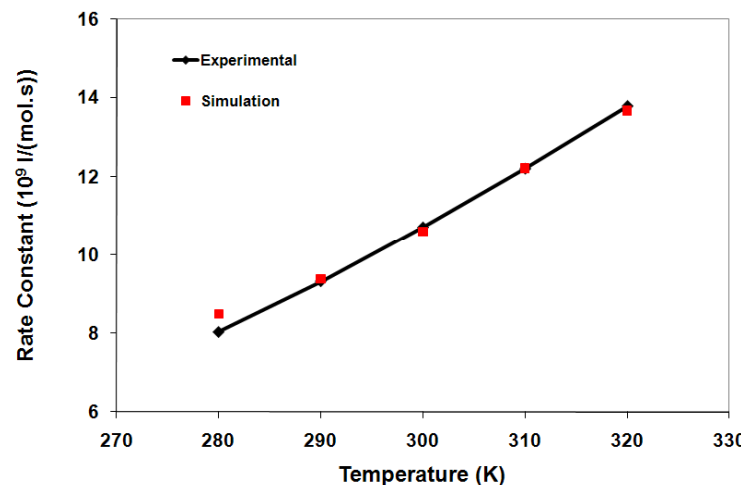
In bulk water, structural diffusivity is about 70% of total diffusivity.

# Proton Transport in Bulk Water

reaction:  $\text{H}_3\text{O}^+ + \text{H}_2\text{O} \rightarrow \text{H}_2\text{O} + \text{H}_3\text{O}^+$   
 rate law:  $\text{rate} = k [\text{H}_3\text{O}^+][\text{H}_2\text{O}]$

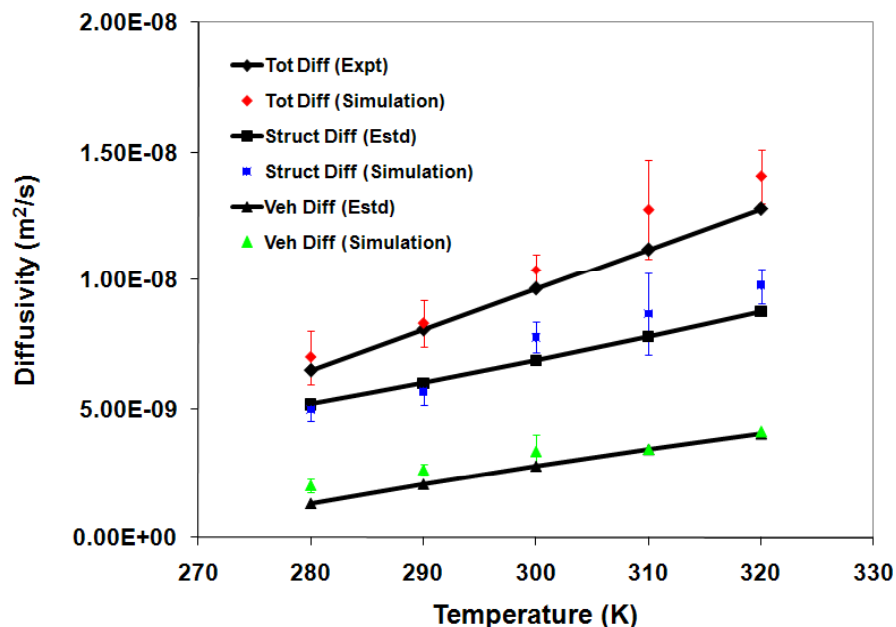
$$k = k_o \exp\left(-\frac{E_a}{RT}\right)$$

- Adjust triggers to fit experimental rate.
- Predict transport properties.



experimental data from  
Luz, Z.; Meiboom, S. J. Am. Chem. Soc., 1964.

**RMD rate constant within 6% of experiment.**



Charge self-diffusivity prediction

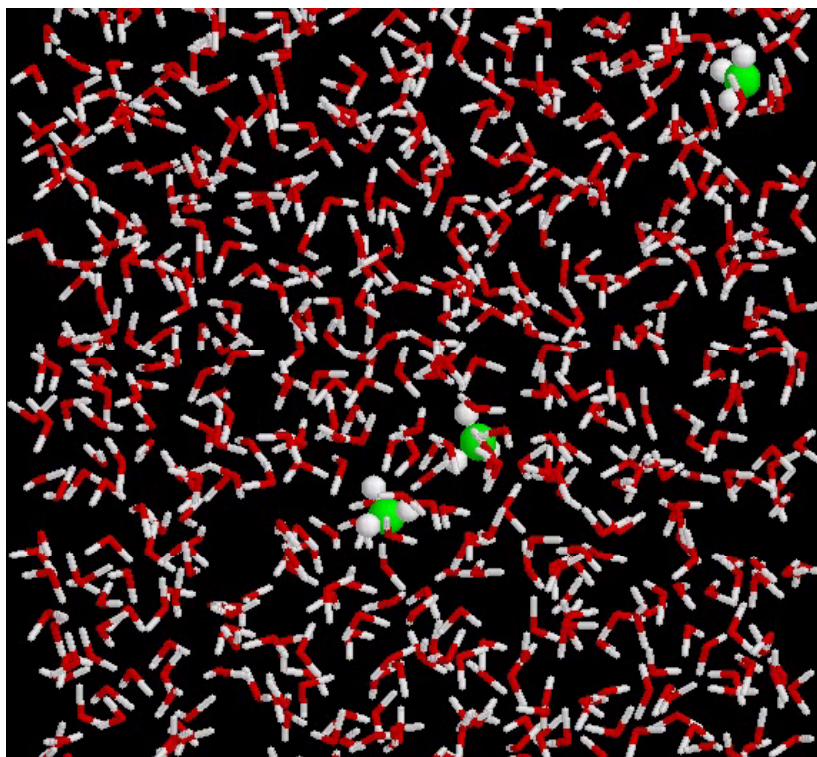
- semi-quantitative agreement with experiment
- decomposition into structural and vehicular components
- structural is 60-70% of total
- correct temperature dependence
- structural and vehicular components are uncorrelated

$$D_{tot} = \lim_{\tau \rightarrow \infty} \frac{\langle \Delta \vec{r}_{veh}^2 \rangle + \langle \Delta \vec{r}_{struct}^2 \rangle + 2 \langle \Delta \vec{r}_{veh} \Delta \vec{r}_{struct} \rangle}{2d\tau}$$



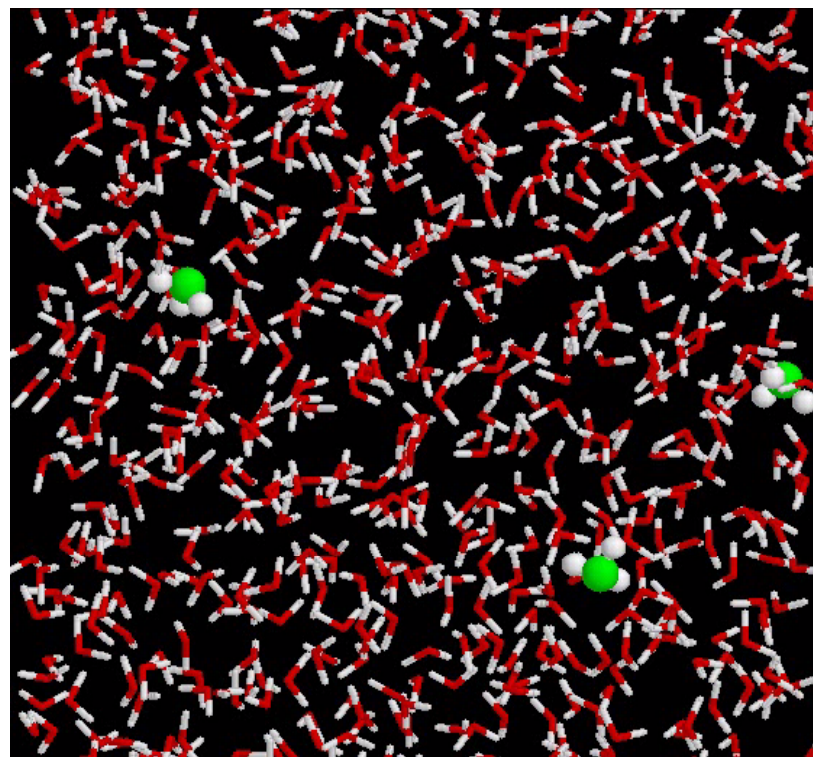
## Proton Diffusion in Bulk Water

Non - Reactive System



Vehicular Diffusion

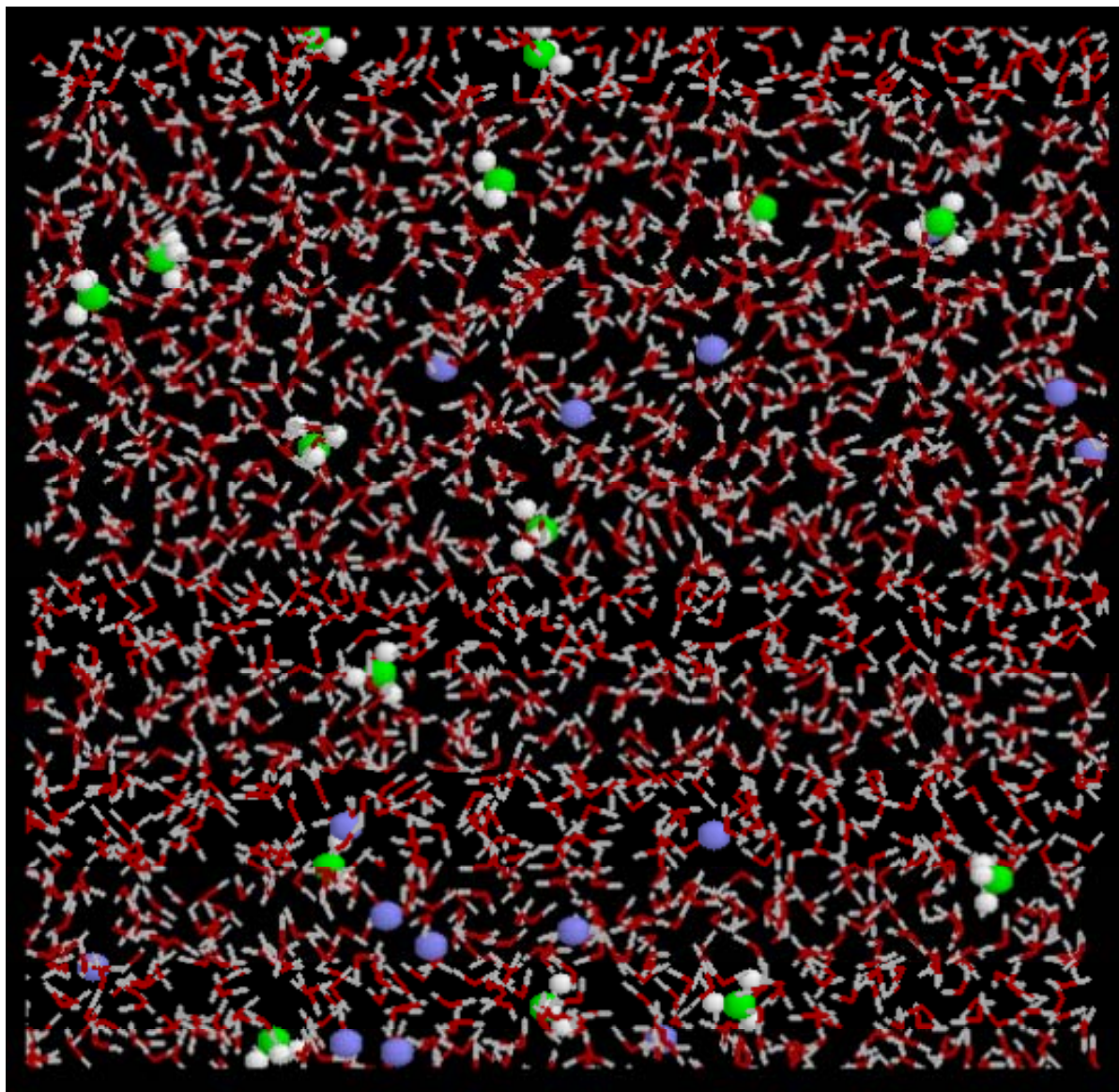
Reactive System



Structural and Vehicular Diffusion

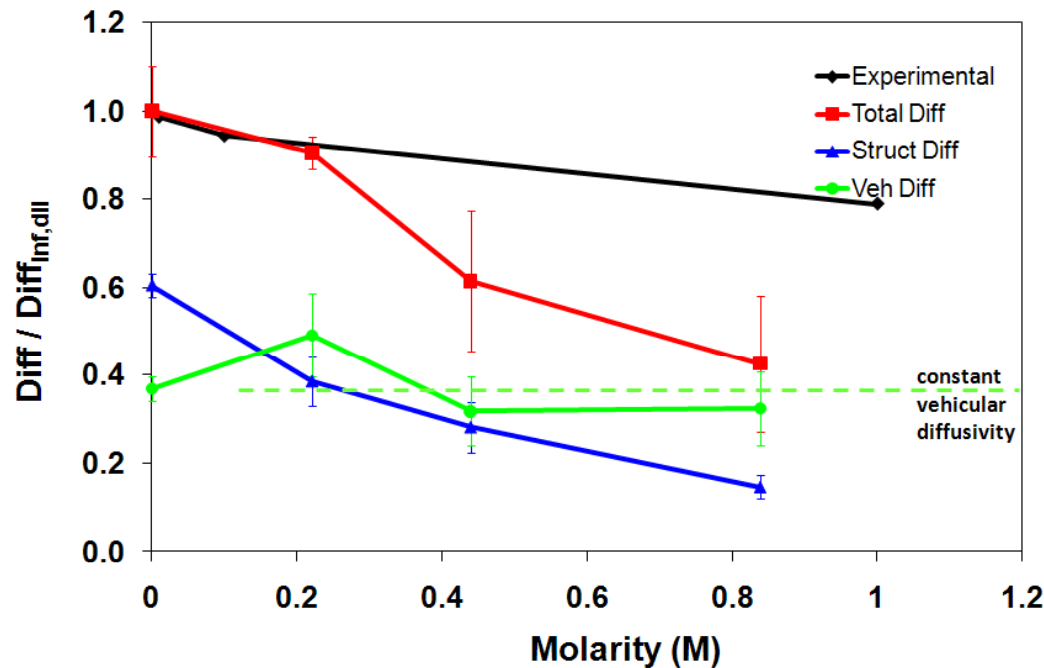
## Bulk HCl Solution: Effect of High Acidity

simulation snapshot  
periodic system  
15  $\text{H}^+$   
15  $\text{Cl}^-$   
1875  $\text{H}_2\text{O}$   
 $\lambda = 125$   
conc = 0.44 M  
pH = 0.36



Legend  
O of  $\text{H}_2\text{O}$  – red  
O of  $\text{H}_3\text{O}^+$  – green  
H – white  
 $\text{Cl}^-$  – blue

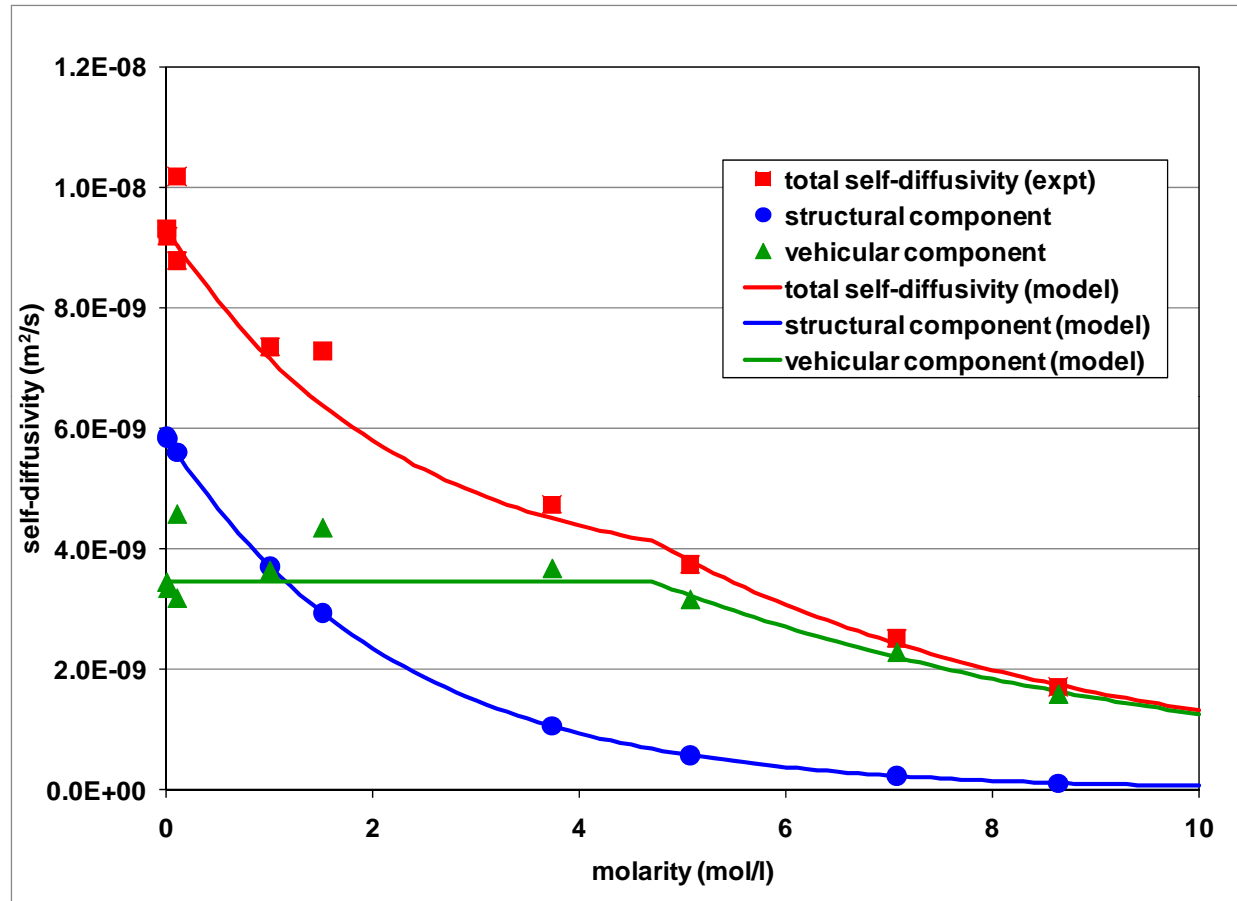
# Bulk HCl Solution: Effect of High Acidity



experimental data from.  
Cornish, B. D.; Speedy, R. J.  
*J. Phys. Chem.*, 1984, 88,  
1888.

- Total charge diffusivity follows the same trend as experimental value but is a bit steeper
- Vehicular component of the charge diffusion is almost constant irrespective of the concentration
- Structural diffusion decreases with increases in HCl concentration and plays a major role in determining the dependence of charge diffusion on the concentration

# Bulk HCl Solution: Effect of Acidity in an Analytical Fit



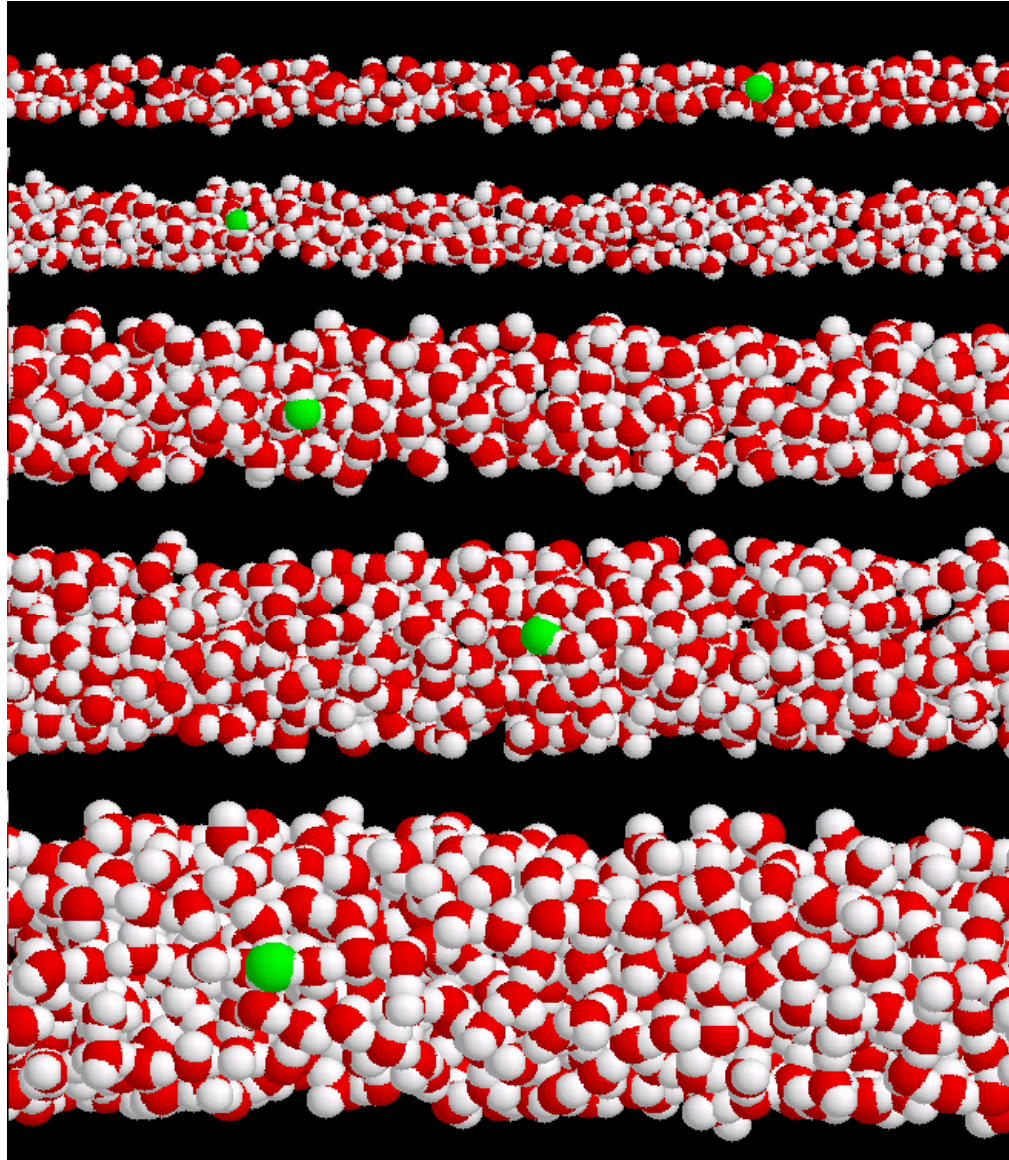
Esai Selvan, M., Calvo-Muñoz, E.M., Keffer, D.J., *J. Phys. Chem. B*, dx.doi.org/10.1021/jp1115004, 2011.

experimental data from.

Cornish, B. D.; Speedy, R. J. *J. Phys. Chem.*, 1984.  
Dippel, Th.; Kreuer, K.D., *Solid State Ionics*, 1991.

- Experimental data for total value
- Two assumptions (validated by RMD) for structural and vehicular components
  - Decline in diffusivity due to pH is in the structural component
  - Structural and diffusive components remain uncorrelated

# Proton Transport in Nanotubes: Effect of Confinement



Nominal radii from 5.42 to 10.85 Å.

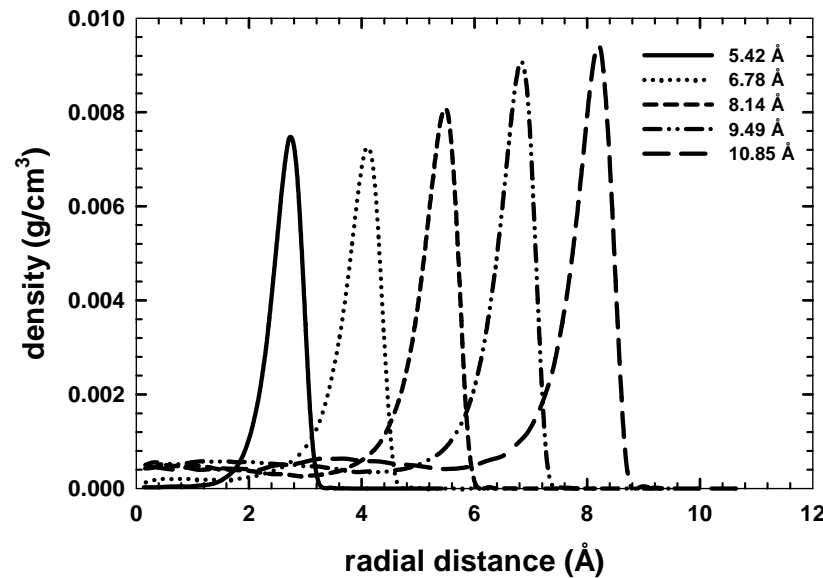
Infinitely dilute simulations (1 excess H<sup>+</sup>)

Results averaged over 144 independent simulations.

Snapshots show H<sub>3</sub>O<sup>+</sup> at pore wall with O atom extended outward.

Esai Selvan, M. *et al.*  
*Mol. Simul.*, 2010.

# Proton Transport in Nanotubes: Effect of Confinement

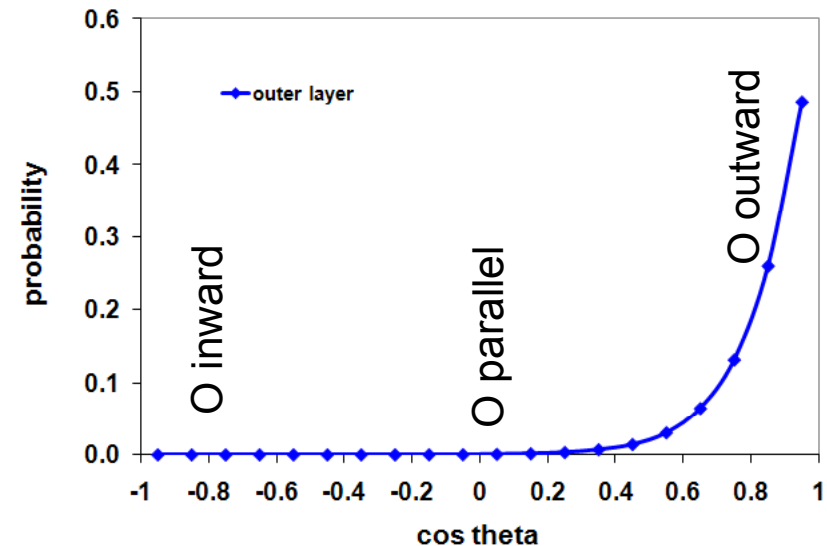


## Density distribution of $\text{H}_3\text{O}^+$ .

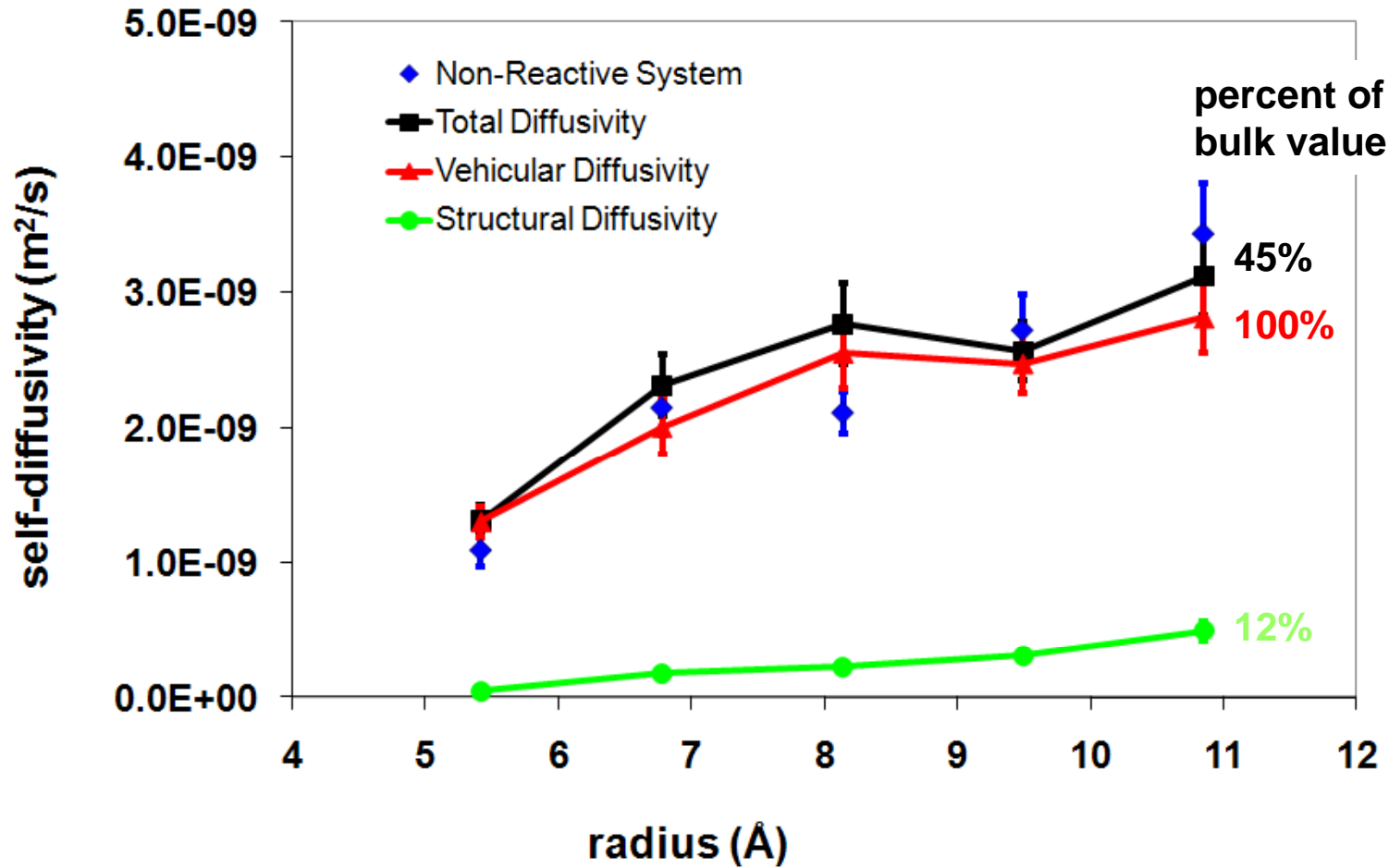
$\text{H}_3\text{O}^+$  is preferentially located at pore wall.

## orientation distribution of $\text{H}_3\text{O}^+$ .

$\text{H}_3\text{O}^+$  is preferentially oriented with oxygen at the pore wall, so as to maximize hydrogen bonding network with 3 hydrogens.



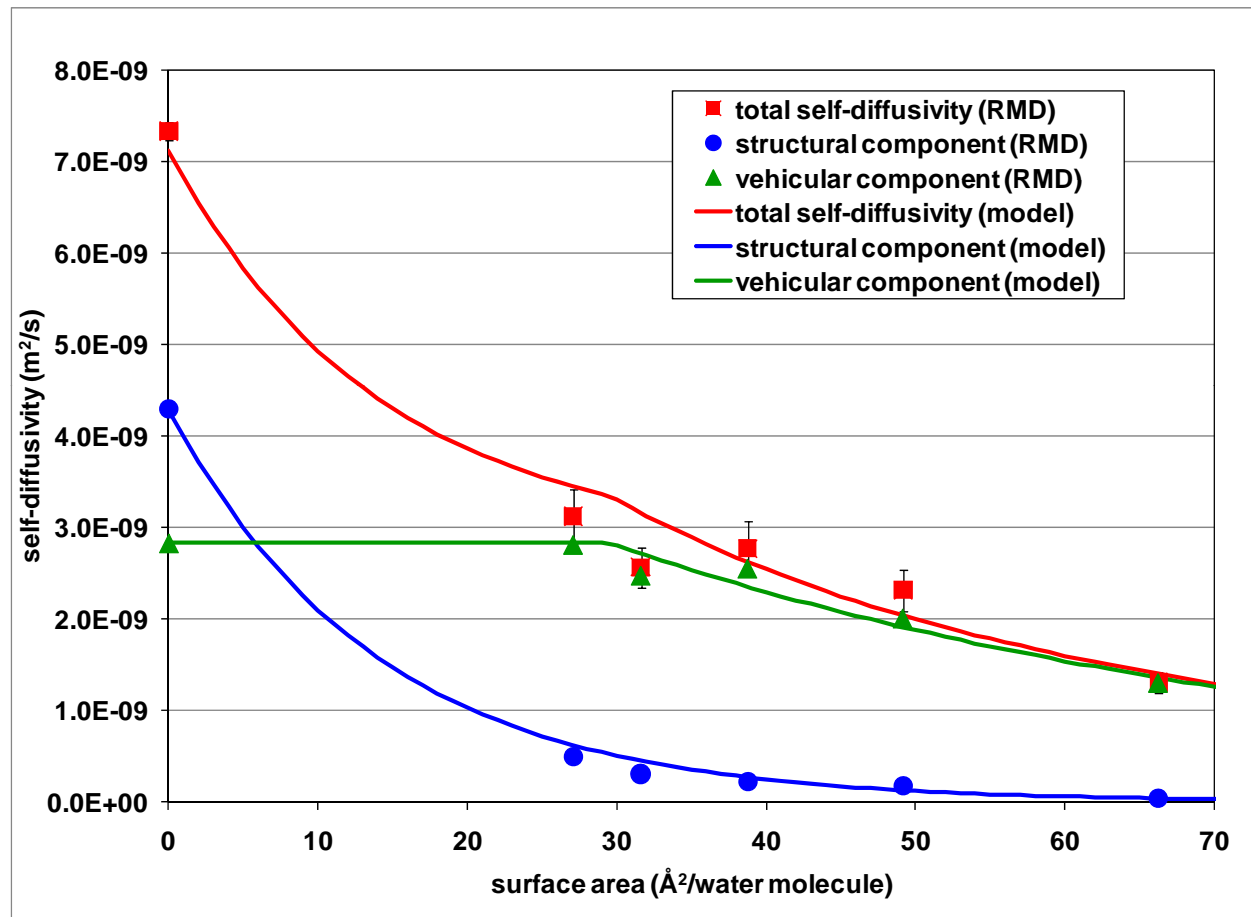
# Proton Transport in Nanotubes: Effect of Confinement



Esai Selvan, M., Keffer, D.J., Cui, S., Paddison, S.J., *36(7-8), Molec. Sim.* pp. 568–578.

Confinement dramatically reduces structural diffusion.

# Nanotubes: Effect of Confinement in an Analytical Fit



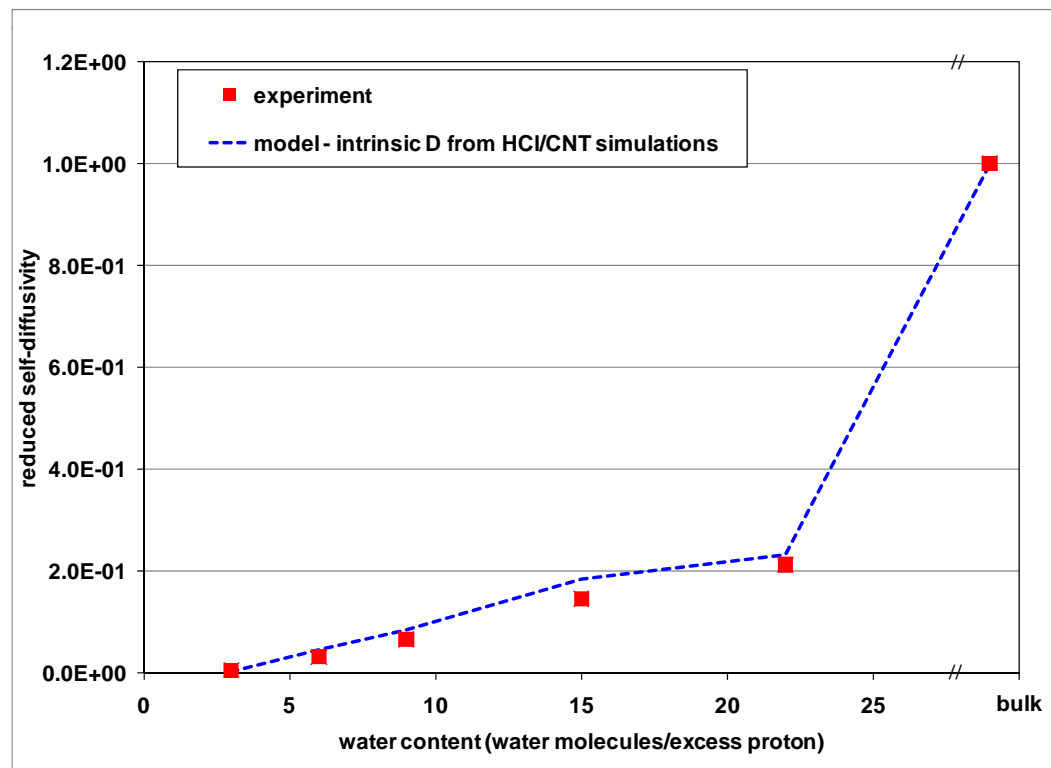
Esai Selvan, M., Calvo-Muñoz, E.M., Keffer, D.J., J. Phys. Chem. B, dx.doi.org/10.1021/jp1115004, 2011.

- Two assumptions (validated by RMD) for structural and vehicular components
  - Decline in diffusivity due to confinement is in the structural component
  - Structural and diffusive components remain uncorrelated



- Acidity – characterized by concentration of  $\text{H}_3\text{O}^+$  in aqueous domain (exponential fit of HCl data)
- Confinement – characterized by interfacial surface area (exponential fit of carbon nanotube data)
- Connectivity – characterized by percolation theory (fit theory to MD/CRW water diffusivity in PEMs)

Esai Selvan, M., Calvo-Muñoz, E.M., Keffer, D.J., J. Phys. Chem. B, dx.doi.org/10.1021/jp1115004, 2011.



Good agreement of theory with experiment.

Theory uses only structural information to predict transport property.

Proton transport is well-described by this simple model.

# Conclusions

Quantum Mechanics calculations provide understanding of structure of ground state and transition state for structural diffusion, activation energy and rate constant

Reactive MD simulations provide molecular-level understanding of coupling of reaction and diffusion in aqueous systems, carbon nanotubes and proton exchange membranes, provides short time mean-square-displacements (MSDs)

Confined Random Walk theory extends MSDs from MD and yield water self-diffusivities in excellent agreement with expt.

An analytical model incorporating

- acidity (concentration of  $\text{H}_3\text{O}^+$  in aqueous domain)
- confinement (interfacial surface area per  $\text{H}_2\text{O}$ )
- connectivity (percolation theory based on  $\text{H}_2\text{O}$  transport)

is capable of quantitatively capturing the self-diffusivity of both water and charge as a function of water content

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Access to the massively parallel machines at Oak Ridge National Laboratory through the UT Computational Science Initiative.



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Vol & Area Analysis



Elisa Calvo-Munoz  
undergraduate  
Confined Random Walks

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